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A fuse control paradigm for demand side management: Formulation and stochastic pricing analysis

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Abstract— **We consider a novel paradigm for demand side management that is based on the so called fuse control concept. We assume that an aggregator communicates with a household only at the meter, imposing a fuse limit, i.e. a restriction in the total consumption level within a given time frame. Consumers are then responsible to adjust the set-points of the individual household devices accordingly to meet the imposed fuse limit. We formulate the problem as a stochastic household management program, with stochasticity arising due to local photovoltaic generation. We show how a demand bidding curve for fuse increments can be constructed as a by-product of the developed problem and provide a rigorous pricing analysis that results in a probabilistic envelope around the "shadow" prices of the deterministic variant of the considered problem, inside which the "shadow" prices of the stochastic one are confined to lie. To evaluate the efficacy of the proposed approach we compare it with an idealized set-up that involves tracking of real-time market price signals.**

I. INTRODUCTION

Power systems are one of the most critical infrastructures in the modern society. To ensure reliable system operation, control services of different nature need to be provided. This task has become more challenging due to the increased level of uncertainty as a result of the increasing penetration of renewable energy sources. To account for this uncertainty not only conventional scheduling and regulation problems need to be revisited, but also conceptually different modeling and control schemes have to be designed.

The conventional approaches involve mainly generation side control. This control method requires adjusting the output of the generators and includes various operational challenges which span different time scales [1]. To account for the intermittent nature of the renewable generation, as well as for other uncertainty sources in the system, research has focused on formulating the stochastic counterparts of the aforementioned problems. Representative work in this context, including stochastic reserve scheduling and unit-commitment can be found in [2–7].

An alternative approach is the so called load side control or demand side management. While controlled loads offer an additional degree of freedom when solving regulation or planning problems in the presence of uncertainty, they should provide a reliable resource to the power network without any disruption of service to the consumers [8], [9]. Toward this direction different approaches for demand side management have been proposed in the literature. Following [9], we can distinguish between price and direct load control. The first approach is based on providing real-time price signals to consumers [10], which will then respond to those signals by appropriately adjusting their consumption level. Due to spikes and volatility in real-time prices, such an approach may expose consumers to price uncertainty and cause discomfort. Direct load control, on the other hand, involves regulating devices and appliances in the household such as thermostatically controlled loads, electric vehicles, water heaters, etc, at an individual or population basis, has attracted significant attention in the literature [11–16].

Another paradigm, that falls between price and direct load control, is referred to as fuse control concept and is discussed in [17]. In this framework an aggregator communicates with a household or a residential area only at the meter, by imposing a restriction on the aggregated consumption level within a specific time frame. Consumers are then responsible to satisfy this limit by appropriately adjusting the set-points of the individual household devices. This approach is less intrusive than direct load control since it enables consumers to meet their contract obligation in many ways that reflect changes in valuation, and does not raise stability issues as in price based control. A conceptually similar work, but following a completely different formulation, is presented in [18], where the authors provide necessary and sufficient conditions for a supply profile to be adequate for meeting an energy requirement (parallel to the fuse constraint considered here) for an aggregation of consumers.

In this paper we focus on the fuse control paradigm as a less intrusive way for direct load control. We formulate the household management problem as a disutility minimization stochastic optimization program subject to a fuse constraint, with stochasticity arising due to local photovoltaic (PV) power generation (load uncertainty can be included similarly). However, we do not consider in this paper storage or load deferral possibilities which is an important aspect of demand response; such extensions are taken into account in [19]. We show that a demand curve for fuse increments, which consumers can disclose to the aggregator, can be constructed as a by-product of the developed problem. This demand curve can then be used by the aggragator to create demand side offers that are bid into the day ahead wholesale market. We provide a rigorous pricing analysis and construct an envelope around the "shadow" prices of the deterministic variant of the proposed problem that bounds the "shadow" prices corresponding to the stochastic problem. To quantify the disutility due to load curtailment when using the constructed curve for bidding in the market, we compare our fuse control approach with a real-time market price set-up.

Section II formulates the household stochastic optimization program arising under the fuse control paradigm. In Section III we show how a demand curve for fuse increments can be constructed and provide a pricing analysis in a stochastic set-up. Section IV includes a simulation based analysis, whereas Section V concludes the paper and provides directions for future work. All proofs have been omitted in the interest of space, but can be found in [19].

II. THE FUSE CONTROL PARADIGM

A. Problem statement

Consider a household or a small residential area comprising N_L uncontrollable loads, N_c controllable loads that will be used to provide demand response services and N_{PV} photovoltaic (PV) generators. Following [20] it is straightforward

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to extend our framework to include storage devices and/or electric vehicles; however, we do not include them here in order to simplify the analysis of Section III, but we show how they can be incorporated in our framework in [19].

We consider a set-up where an aggregator interacts with the household only at the household meter. The household is subscribed to a given fuse limit, that is imposed by the aggregator and is activated remotely in case the total net load in the household should be limited to that level to ensure the well functioning of the entire network. Once this limit is imposed, the household is responsible to optimize the schedule of the individual devices in the most cost efficient way by adjusting their set-points, while the aggregator is not involved in this process. The fuse limit may be contingent on some exogenous variables defined in the service contract.

Let N denote an optimization horizon with unitary steps. Every T steps a fuse limit is imposed, representing a budget constraint that requires the total net load in the household not to exceed this limit. Assume that N, T are such that N/T is an integer, and let $\{P_f(i)\}_{i=1}^{N/T} \in \mathbb{R}^{N/T}$ denote the fuse profile for the optimization horizon. Moreover, for each $k = 1, \dots, N$, $j = 1, \dots, N_c$, $P_L^j(k) \in \mathbb{R}$ denotes the power of the uncontrollable load j in time-step k, which is treated as a constant in our analysis but can be easily modelled as an uncertain variable. Similarly, $P_c^j(k) \in \mathbb{R}$ denotes the power of the controllable load j in time-step k.

For $k = 1, \dots, N$, $j = 1, \dots, N_{PV}$, let $P_{PV}^j(k)$ represent the PV power forecast of generator j in time-step k. Since forecasts are in general inaccurate, we will perform a stochastic analysis, taking forecast errors into account. For each $j = 1, \dots, N_{PV}$, let $\delta^j = (\delta^j(1), \dots, \delta^j(N))$ be a vector including the forecast error of each PV unit. Moreover, let $\delta = (\delta^1, \dots, \delta^{N_{PV}}) \in \Delta$ be distributed according to an absolutely continuous distribution $\mathbb{P}$, where Δ is a compact, possibly infinite, set. This distribution may be unknown, but we assume that we are able to extract, or we are provided with, samples from this distribution (e.g. historical data). The continuity assumption is only needed in the proof of Theorem 1. Since all forecast errors for the individual units and the different time-steps, are collectively included in δ, spatial and temporal correlation is respected once a sample is extracted from Δ according to $\mathbb{P}$.

We treat the fuse control paradigm as a disutility minimization problem, where the objective is to find the optimal dispatch for the household loads that minimizes the deviation from a baseline load profile, which is assumed to be fixed (e.g. it may correspond to the solution of the deterministic variant of the problem). Specifically, we seek to determine a load dispatch policy that minimizes $\sum_{k=1}^{N} \sum_{j=1}^{N_c} \mathcal{R}_{\delta\in\Delta}[U^j(k,\delta)]$, where $\mathcal{R}_{\delta\in\Delta}[\cdot]$ is any given risk metric. For example, it can represent the expected value of its argument or its worst case value (take $\mathcal{R}_{\delta\in\Delta}[\cdot] = \sup_{\delta\in\Delta} ||\cdot||$, where $||\cdot||$ is the first or the Euclidean norm). For $k = 1, \dots, N$, $U^j(k,\delta) \in \mathbb{R}$ denotes the disutility of load $j = 1, \dots, N_c$. We consider $U^j(k,\delta) = \rho^j(k)\big(P_{c,\text{base}}^j(k) - P^j(k,\delta)\big)$, to penalize the deviation of the load dispatch policy $P^j(k,\delta)$, whose structure will be defined next, from a baseline load level $P_{c,\text{base}}^j(k) \in \mathbb{R}$, i.e. we assign a penalty to load curtailment. Coefficient $\rho^j(k) \in \mathbb{R}_+$ is a penalty factor, possibly different for each $j = 1, \dots, N_c$, $k = 1, \dots, N$. Notice that, even not shown explicitly, $U^j(k,\delta)$ depends on the decision variables $P_c^j(k), d_+^j(k), d_-^j(k)$ that constitute the dispatch policy $P^j(k,\delta)$ and will be defined in the sequel.

We thus have the following family of problems, parameterized by the uncertainty set and the fuse profile, and we will refer to each of them as $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$.

$$\min_{\left\{\left\{P_c^j(k),\ d_+^j(k),\ d_-^j(k)\right\}_{j=1}^{N_c}\right\}_{k=1}^{N}} \sum_{k=1}^{N}\sum_{j=1}^{N_c} \mathcal{R}_{\delta\in\Delta}[U^j(k,\delta)] \quad (1)$$

subject to:
1) Fuse limit: For each $i = 1, \dots, N/T$ the total net load in the household should be restricted to the corresponding element of the fuse profile, for all $\delta \in \Delta$, i.e.

$$\sum_{k=iT-T+1}^{iT} \Big[\sum_{j=1}^{N_L} P_L^j(k) - \sum_{j=1}^{N_{PV}} \big(P_{PV}^j(k) + \delta^j(k)\big) + \sum_{j=1}^{N_c} P^j(k,\delta)\Big] \leq P_f(i),\ \forall \delta \in \Delta, \quad (2)$$

where $P^j(k,\delta) \in \mathbb{R}$ is the sum of a deterministic component which is the dispatch $P_c^j(k)$ of the controllable loads, and two terms that depend on the uncertainty error and are mutually exclusive. This is encoded by

$$P^j(k,\delta) = P_c^j(k) + d_+^j(k) \max\big(0, \sum_{\ell=1}^{N_{PV}} \delta^\ell(k)\big) - d_-^j(k) \max\big(0, -\sum_{\ell=1}^{N_{PV}} \delta^\ell(k)\big). \quad (3)$$

Note that $P_c^j(k)$ can be thought of as a first stage decision, whereas $d_+^j(k),\ d_-^j(k) \in \mathbb{R}$ can be thought of as the coefficients of an affine recourse action. In particular, the stochastic terms imply that if an uncertain error is realized, it should be allocated to the controllable loads according to the coefficients $d_+^j(k),\ d_-^j(k)$, adjusting their set-point $P_c^j(k)$. If the total forecast error is positive, the loads should increase their power consumption, while if it is negative they should decrease it. To encode this error allocation protocol we impose the following set of constraints on the coefficients $d_+^j(k),\ d_-^j(k)$.

2) Allocation constraints: For each $k = 1, \dots, N$, the allocation coefficients should satisfy

$$\sum_{j=1}^{N_c} d_+^j(k) = 1,\ \sum_{j=1}^{N_c} d_-^j(k) = 1,\ d_+^j(k), d_-^j(k) \geq 0, \quad (4)$$

which imply that they should be positive and sum up to one. The positivity of the allocation coefficients is required only in the proof of Proposition 2. If constructing a "shadow" price envelope is not desirable we could allow the allocation coefficients to be also negative, since this may lead to more profitable solutions for some choices of the objective function.

3) Controllable load limits: For each $k = 1, \dots, N$, $j = 1, \dots, N_c$, the set-point of each load together with its adjustment in case of a forecast error should satisfy

$$\alpha^j(k) P_{c,\text{base}}^j(k) \leq P^j(k,\delta) \leq P_{c,\text{base}}^j(k),\ \forall \delta \in \Delta, \quad (5)$$

where $P^j(k,\delta)$ is given by (3), $P_{c,\text{base}}^j(k) \in \mathbb{R}$ is a given baseline load level and $\alpha^j(k) \in [0,1]$ characterizes the flexibility margins of each load. We only curtail loads from the baseline profile; flexibility in exceeding the baseline load consumption can be modeled analogously.

Problem $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$, given by (1)-(5), is a constrained minimization problem. Since we do not have coupling constraints between consecutive T time-steps, we could consider $N = T$ and solve the optimization problem for every T steps in parallel. This can also simplify the averaging procedure of Section III-A; this is not the case, however, if storage dynamics are included in the formulation.

B. Problem reformulation

By inspection of (2), (4), it can be shown that $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ is equivalent to a problem that involves minimizing the same objective function (1) subject to (4), (5), but with the uncertain fuse constraint being replaced by a deterministic constraint.

Proposition 1: $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ is equivalent to a problem that involves minimizing (1) subject to (4), (5) and

$$\sum_{k=iT-T+1}^{iT} \Big[\sum_{j=1}^{N_L} P_L^j(k) - \sum_{j=1}^{N_{PV}} P_{PV}^j(k) + \sum_{j=1}^{N_c} P_c^j(k) \Big] \leq P_f(i). \tag{6}$$

To prove Proposition (1), it suffices to show that (6) emanates from (2), (4). Substituting (3) in (2), and after some algebraic manipulations the result follows. The introduction of the allocation coefficients is inspired by the analysis of [6], where such coefficients were introduced to allocate the generation-load mismatch among the various generating units. Proposition 1 parallels the fact that in [6] only a deterministic power balance constraint has to be imposed.

$\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ is then given by (1)-(5), with (2) replaced by (6). However, (5) should be satisfied for all $\delta \in \Delta$. Δ may be an infinite set, rendering $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ a semi-infinite optimization program, which is not easy to solve in general. Therefore, we relax (5) and impose the load limit constraints not for every $\delta \in \Delta$, but for any $\delta \in S_m = \{\delta_i\}_{i=1}^m$, where $S_m \subset \Delta$ is a discrete set containing m identically and independently distributed realizations of the uncertainty error. This gives rise to a finite dimensional linear program. Due to the decoupled structure of the problem, for any $k = 1, \ldots, N$, it suffices to enforce (5) only for the extreme values of $\sum_{\ell=1}^{N_{PV}} \delta^\ell(k)$, among the samples in S_m. Moreover, the risk metric is substituted by $\mathcal{R}_{\delta \in S_m}[\cdot]$.

The resulting family of optimization programs can be denoted as $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$. We assume throughout the paper that $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ is feasible, its feasibility region has a non-empty interior and it admits a unique optimal solution. Fix ϵ, $\beta \in (0,1)$ and extract $m \geq \frac{e}{e-1}\frac{1}{\epsilon}\big(N_{PV}N - 1 + \ln\frac{1}{\beta}\big)$ samples to construct S_m. Following [21], [22], with confidence at least $1-\beta$, the minimizer of $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ satisfies (3)-(6) with probability at least $1-\epsilon$. This implies that we can accompany our solution with an a-priori (probabilistic) certificate regarding the satisfaction of the system constraints.

III. Pricing analysis

A. Demand curve for fuse increments

In Section II-B we formulated a family of problems $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ parameterized by S_m and $\{P_f(i)\}_{i=1}^{N/T}$. For any given fuse profile, $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ provides the load dispatch that minimizes the total load disutility. For each $i = 1, \ldots, N/T$, the dual variable associated with each constraint in (2) shows the effect in the disutility of an incremental change in $P_f(i)$. Let $\lambda[S_m, P_f(i)] \in \mathbb{R}_+$ denote this dual variable. Variable $\lambda[S_m, P_f(i)]$ should be interpreted as a "shadow" price.

This intuitive interpretation is due to the fact that we have uncertainty independent fuse limit constraints, as an effect of the use of the allocation vectors. This is in contrast to other stochastic scheduling approaches that introduce a different set of decision variables (increasing also the computational burden) and enforce different constraints in the form of (6) per uncertainty sample. This results in a different dual variable per sample and it is then unclear which of them (or their expected value) should be selected as "shadow" price.

We aim at constructing a demand curve that the household will reveal to the aggregator, who will use it to bid in the day-ahead market. Consider the vector $\bar{P}_f \in \mathbb{R}^{N_f}$ containing a finite number of values that $P_f(i)$ may take as an effect of some discretization process, and construct the $N_f^{N/T}$ different fuse profiles that may occur. For each of them we solve $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ and record $\{\lambda[S_m, P_f(i)]\}_{i=1}^{N/T}$. For each distinct value of $\bar{P}_f(i)$, average among the recorded dual variables that correspond to this fuse value and denote by $\bar{\lambda}[S_m, \bar{P}_f(i)]$ the resulting averaged dual variable. Since every T steps are decoupled, it suffices to consider here only the N_f profiles for which the fuse limit is constant across the optimization horizon to $\bar{P}_f(i)$, $i = 1, \ldots, N_f$.

The quantity $\bar{\lambda}[S_m, \bar{P}_f(i)]$ is based on the forecast and PV power error values for a given optimization horizon. We can repeat the entire process for different PV power forecasts and error realizations, and then compute the average among all $\bar{\lambda}[S_m, \bar{P}_f(i)]$. With a slight abuse of notation, in the sequel we use the same symbol $\bar{\lambda}[S_m, \bar{P}_f(i)]$ to represent the resulting average quantity. Note that there are two different averaging procedures involved: the first is when constructing $\{\bar{\lambda}[S_m, \bar{P}_f(i)]\}_{i=1}^{N_f}$ from $\{\lambda[S_m, P_f(i)]\}_{i=1}^{N/T}$, and the second is when averaging among the "shadow" prices of problems that correspond to different PV power forecasts (e.g. different representative days).

Therefore, $\bar{\lambda}[S_m, \bar{P}_f(i)]$, corresponds to an average "shadow" price according to which the aggregator will bid for supplying load reduction in the wholesale day ahead market. Having $\bar{\lambda}[S_m, \bar{P}_f(i)]$ as a function of $\bar{P}_f(i)$, $i = 1, \ldots, N_f$, and for the numerical values of Section IV, we can compute the demand bidding curve as shown in Fig. 1 in "red", which as expected is non-increasing. Notice that, due to the complementarity slackness optimality condition for $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$, having non-zero "shadow" prices implies that the corresponding fuse limit constraints are binding. This is due to the structure of $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$.

B. Stochastic vs. deterministic "shadow" prices

The "shadow" price $\bar{\lambda}[S_m, \bar{P}_f(i)]$, $i = 1, \ldots, N_f$, is related to the dual variables associated with the fuse constraints (2). Even though these constraints are deterministic (Proposition 1), the dual variables depend on the uncertainty since (2), (5) are coupled through the decision variables. Therefore, if we consider the deterministic counterpart $\mathcal{P}[\emptyset, \{P_f(i)\}_{i=1}^{N/T}]$ of $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$, we get different "shadow" prices $\{\bar{\lambda}[\emptyset, \bar{P}_f(i)]\}_{i=1}^{N_f}$, and hence a different demand curve. This deterministic curve is shown with "blue" in Fig. 1.

Determining how different the stochastic and the deterministic curves are is not straightforward. We construct an envelope around the deterministic curve, inside which the stochastic one is confined to lie. To this end, let S_m^+ be constructed from S_m such that for any sample $\delta_i \in S_m$, $i = 1, \ldots, m$, any element $\delta_i^j(k)$ of δ_i is replaced by $\max(0, \delta_i^j(k))$. Define S_m^- similarly, with $\delta_i^j(k)$

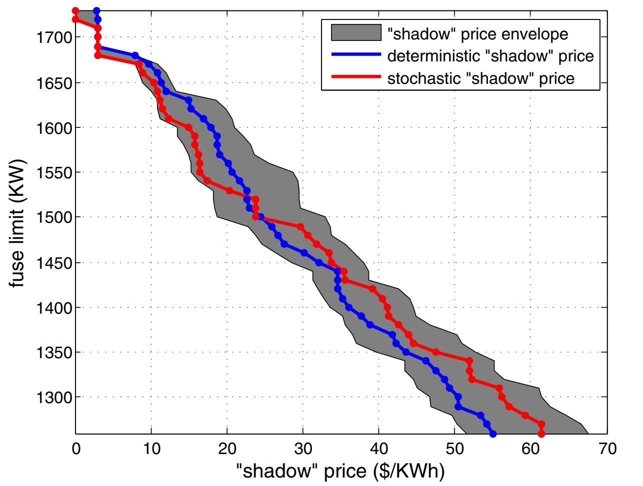

Fig. 1. Demand curve for the stochastic problem "red"; demand curve for the deterministic problem "blue"; Demand curve envelope "gray", inside which the deterministic and stochastic curves are confined to lie.

replaced by $\min(0, \delta_i^j(k))$. That way, S_m^+, S_m^- have only non-negative and non-positive elements, respectively. Let $\mathcal{P}[S_m^+, \{P_f(i)\}_{i=1}^{N/T}]$ and $\mathcal{P}[S_m^-, \{P_f(i)\}_{i=1}^{N/T}]$ be the corresponding dispatch problems and $\bar{\lambda}[S_m^+, \bar{P}_f(i)]$, $\bar{\lambda}[S_m^-, \bar{P}_f(i)]$ the associated "shadow" prices computed according to the averaging procedure of the previous subsection, when S_m is substituted with S_m^+ and S_m^-, respectively.

Proposition 2: For any $m \in \mathbb{N}_+$ and $i = 1, \ldots, N_f$,

$$1)\ \bar{\lambda}[\emptyset, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big], \tag{7}$$

$$2)\ \bar{\lambda}[S_m, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]. \tag{8}$$

This is a monotonicity statement, showing that if we expect that the uncertainty error will only increase ($\delta \in S_m^+$) or will only decrease ($\delta \in S_m^-$), then the demand curve should be shifted towards the left and right, respectively. If S_m^+ or S_m^- is empty, then the corresponding "shadow" price coincides with the one of the deterministic problem. The price envelope is depicted in Fig. 1 and its boundaries correspond to the cases where $\delta \in S_m^+$ and $\delta \in S_m^-$ (if the uncertainty error was bounded, these boundaries would correspond to the error extrema). Since "shadow" prices depend on the uncertainty, the computed envelope shows how an uncertainty error is translated in the "shadow" price domain. Whether $\bar{\lambda}[S_m, \bar{P}_f(i)]$ is lower or higher than $\bar{\lambda}[\emptyset, \bar{P}_f(i)]$ depends on the maximum value of $\rho^j(k)$ among the (j,k), $k \in [iT - T + 1, iT]$, indices that correspond to inactive constraints.

A direct consequence of the proof of Proposition 2, is that the "shadow" price in the stochastic set-up will be equal to the "shadow" price of the deterministic one only if the maximum value admitted by $\rho^j(k)$ among the (j,k) indices of the inactive constraints is the same in both problems. This is due to the fact that the dual variable of each fuse constraint is equal to the maximum value attained by the penalty factor $\rho^j(k)$ among all (j,k) indices that correspond to inactive constraints (see proof of Proposition 2). Our analysis depends on the structure of (5), where the uncertainty appears multiplied by the allocation coefficients (see (3),(5)), which all have the same sign due to (4); the validity of these results for other problem structures needs further investigation.

The economic interpretation is that if the error is expected to be non-negative (similarly for negative error), the total power consumption level is expected to increase, and hence the prices will be lower, as if we had a problem with a fuse limit higher by the amount of the forecast error. Then the expectation about the evolution of prices changes compared to the deterministic case. Therefore, for a given "shadow" price, the consumers are willing to purchase a lower quantity, leading to a shift in the demand curve towards left.

To generalize this statement we quantify the probability with which the computed envelope remains unchanged if a new sample δ is realized. Consider $\mathcal{P}[S_m \cup \{\delta\}, \{P_f(i)\}_{i=1}^{N/T}]$ and let $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)]$, be the associated "shadow" prices. To simplify the statement of the following theorem, assume that the aforementioned "shadow" prices correspond to the exact dual variables and are not average quantities.

Theorem 1: Assume that $\mathbb{P}$ is any absolutely continuous probability measure. Fix ϵ, $\beta \in [0,1]$. If $m \geq \frac{e}{e-1}\frac{1}{\epsilon}\big(N_{PV}N - 1 + \ln\frac{1}{\beta}\big)$, then for all $i = 1, \ldots, N_f$, with confidence at least $1 - \beta$, $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]$ with probability at least $1 - \epsilon$, i.e.

$$\mathbb{P}^m\Big[(\delta_1, \ldots, \delta_m) \in \Delta^m : \mathbb{P}\Big[\delta \in \Delta : \bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]\Big] \geq 1 - \epsilon\Big] \geq 1 - \beta. \tag{9}$$

$\mathbb{P}^m$ denotes the product probability measure. Note that the number of samples that need to be extracted for (9) to hold, depends linearly on the total number of uncertainty variables $N_{PV}N$ due to [21]. Other sample size bounds can be used as well [23], [22]. If $\bar{\lambda}[S_m^+, \bar{P}_f(i)]$, $\bar{\lambda}[S_m^-, \bar{P}_f(i)]$ were average quantities, computed based on a finite number of "shadow" prices, the result of Theorem 1 would hold with the following modification: Since for every individual "shadow" price that contributes in the average, (9) would be satisfied with possibly different ϵ and β, (9) would also hold for the average "shadow" prices with ϵ and β replaced by the sum of the individual ϵ, β.

IV. Simulation Study

A. Simulation set-up

We consider the problem described in Section II with $N_L = 3$, $N_c = 5$ and $N_{PV} = 1$. The planning horizon was chosen to be $N = 32$ and we assumed that the fuse limit is communicated every $T = 4$ steps. Every step of the planning horizon corresponds to a 15 minute interval, which implies that the fuse profile has granularity of one hour. The risk metric in (1) was chosen to be the worst case metric based on the first norm. For the sake of this study, we selected $\rho^j(k)$ from a uniform distribution in the interval $[0, 90]$. To compute $\bar{\lambda}[S_m, \bar{P}_f(i)]$, $i = 1, \ldots, N_f$, we selected $N_f = 48$ values starting from 1250KW with granularity of 10KW, and averaged (see discussion in Section III-A) across four representative days in the course of one summer month. For each day the forecast ("red") and the forecast plus errors ("gray") are shown in Fig. 2. The forecast values correspond to normalized data taken from [10]. The PV power output is zero during the hours of no irradiation.

Following [24], to generate forecast error time series for the PV power output, we simulated the stochastic process

$$\delta^j(k+1) = \max\big(\delta^j(k) + u^j(k), -P_{PV}^j(k+1)\big), \tag{10}$$

with $\delta^j(1) = 0$, until the time-step that corresponds to the peak power production in Fig. 2. After that time, we assumed that the error follows a mirrored pattern so that it degrades until the time of zero production. Variable $u^j(k)$ is extracted from a normal distribution with zero mean

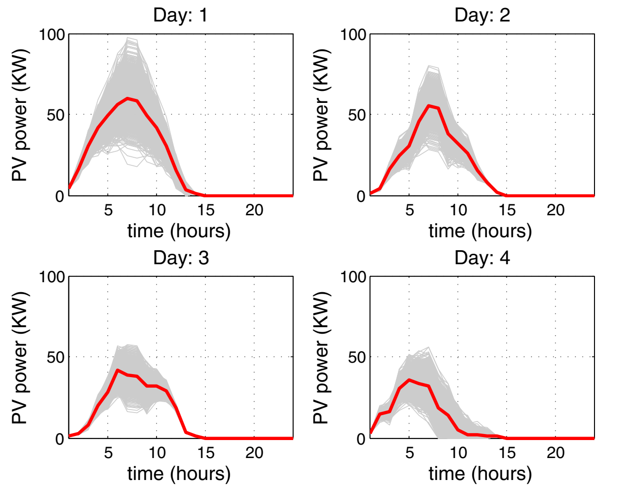

Fig. 2. PV power output for four representative days. For each day, the "red" curve corresponds to the forecast, whereas the "gray" curves show the forecast plus the forecast errors.

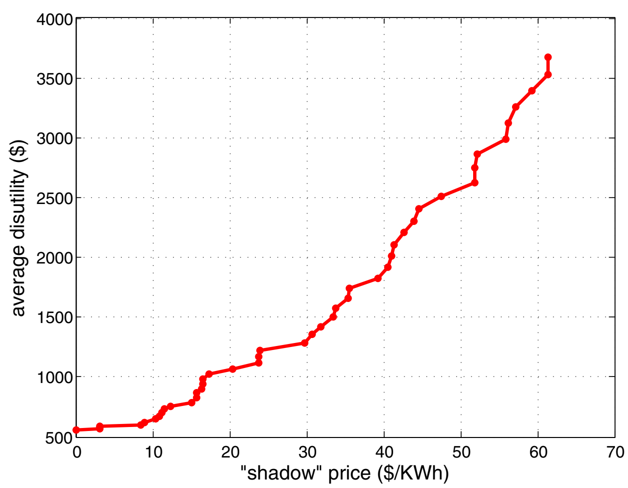

Fig. 3. Average disutility due to load curtailment vs. "shadow" price.

and $0.1P_{PV}^j(k)$ standard deviation. In case of multiple PV units, $u^j(k)$, $j = 1, \ldots, N_{PV}$, could be extracted from a multivariate normal distribution to take into account spatial correlation as well. The $\max(\cdot,\cdot)$ operator ensures that the forecast power plus the generated error does not take negative values. A more involved time-series model is outside the scope of this paper. The number of samples we generated was according to Theorem 1, for $\epsilon = 0.03$ and $\beta = 10^{-11}$.

All simulations were carried out using the solver LINPROG (all the resulting optimization problems are linear programs) under the MATLAB interface YALMIP [25].

B. Comparison with a real-time market price set-up

The demand curve constructed in Section III-A can be used by the aggregator to bid in the day-ahead market. For a given day-ahead market price signal (with granularity of one hour), the disutility due to load curtailment (i.e. $\sum_{k=1}^{N}\sum_{j=1}^{N_c}\rho^j(k)\big(P_{c,\text{base}}^j(k) - P_c^j(k)\big)$) for each hour is related to the point on the vertical axis in Fig. 1 that corresponds to this price value along the "red" curve. This point is a specific value of the fuse limit. Since $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ was parametric with respect to the fuse profile, similarly to the way we computed the average prices $\{\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)]\}_{i=1}^{N_f}$, we can compute the average disutility that corresponds to each $\bar{P}_f(i)$, $i = 1, \ldots, N_f$, and then perform linear interpolation to compute the disutility and any specific fuse limit. Following this procedure we can construct the curve shown in Fig. 3, which shows the average disutility that corresponds to each of the "shadow" prices in Fig. 1. As expected, the higher the "shadow" price, the higher is the disutility due to load curtailment.

To compare the disutility due to load curtailment that occurs when using the fuse control paradigm, we used as a benchmark a set-up where the loads in the household respond directly to real-time market prices. To formulate this real-time price tracking problem, consider the following deterministic optimization program:

$$\mathcal{P}_{RT}: \min_{\left\{\left\{P_c^j(k)\right\}_{j=1}^{N_c}\right\}_{k=1}^{N}} \sum_{k=1}^{N}\sum_{j=1}^{N_c}\Big(\mu(k)P_c^j(k) + U^j(k,0)\Big) \tag{11}$$

subject to

$$\alpha^j(k)P_{c,\text{base}}^j(k) \leq P_c^j(k) \leq P_{c,\text{base}}^j(k),$$
$$\forall j = 1, \ldots, N_c,\ \forall k = 1, \ldots, N. \tag{12}$$

Note that the second term in (11) corresponds to the load disutility evaluated at $\delta = 0$, unlike the objective function of $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ where a risk metric was employed due to the presence of uncertainty. Constraint (12) is the deterministic variant of (5). Parameter $\mu(k)$ corresponds to the value of the real-time market price signal at time-step k. Once problem $\mathcal{P}_{RT}$ is solved, we can compute the disutility $\sum_{j=1}^{N_c} U^j(k,0)$, $k = 1, \ldots, N$, evaluated at the resulting optimal solution. By inspection of $\mathcal{P}_{RT}$, some fraction of load j will be curtailed at time-step k only if $\mu(k) > \rho^j(k)$.

The real-time market price signal used corresponds to normalized data, taken from [26] for the period 1-28 May 2014 with granularity of 15 minutes. The day-ahead market price signal is constructed by averaging accross the real-time one for the same period but for different years; since we are interested in the disutility per hour we also averaged among the intra-hour values to determine an hourly profile. The market price signals are shown in Fig. 4; the "red" curve indicates the day-ahead market price signal and the "gray" curve the real-time one. Fig. 5 shows the resulting disutility due to load curtailment for the fuse control paradigm ("red") and the real-time price set-up ("gray"). The expected disutility (averaged accross all hours of the price profiles) is $1,042.2$\$ for the fuse control paradigm and 912.8\$ for the case where real-time prices are employed, i.e. 14.2% higher disutility. The difference in disutility between the two approaches is a measure for the efficiency loss due to the fuse control concept. A moderate difference implies that the proposed approach offers an efficient alternative to real-time price control, without raising stability issues as in real-time pricing, and while offering the consumers the possibility to select a demand response contract (see Section IV-B of [19]). Notice that the disutility due to load curtailment follows closely the market price patterns of Fig. 4.

For the rest of this subsection we validate, for a confidence level $\beta = 10^{-11}$, the statement in (9) empirically (without averaging the "shadow" prices; see also discussion below Theorem 1). To achieve this, for a given S_m we performed 10,000 Monte Carlo simulations corresponding to different realizations $\delta \in \Delta$. For each $i = 1, \ldots, N_f$, the empirical probability that $\bar{\lambda}[S_m \cup$

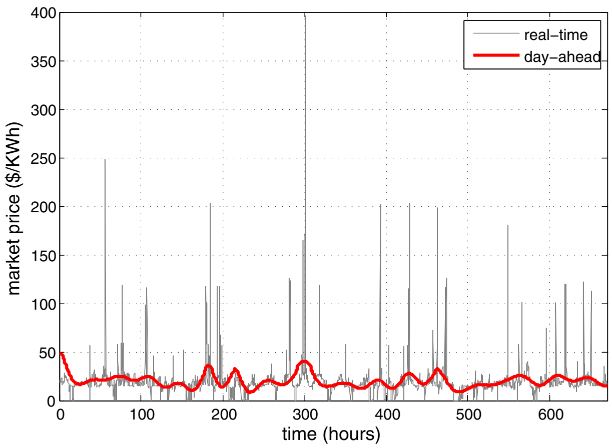

Fig. 4. Day-ahead ("red") and real-time ("gray") market price signals for the period 1-28 May 2014.

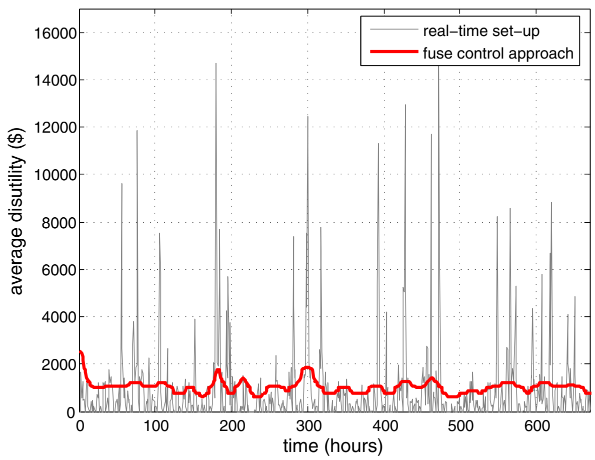

Fig. 5. Average disutility due to load curtailment for the fuse control ("red") and real-time ("gray") price set-up.

$\{\delta\}, \bar{P}_f(i)] \in [\bar{\lambda}[S_m^+, \bar{P}_f(i)], \bar{\lambda}[S_m^-, \bar{P}_f(i)]]$ can be then computed as the number of simulations out of the 10,000 runs for which the inclusion constraint is satisfied. We found out that this empirical estimate is 0.987 (it turned out that for this set-up this is the same for all $i = 1, \ldots, N_f$), which is higher compared to the theoretical value $1 - \epsilon = 0.97$, implying that the bound in (9) is conservative.

V. Conclusion

In this paper we considered the fuse control paradigm for demand side management. We formulated this problem as a stochastic optimization program, and conducted a probabilistic pricing analysis. The pricing analysis presented here was related to load side management, but it is also applicable to generation dispatch problems without network constraints. Current focus is to investigate the validity of our pricing analysis in a stochastic, nodal pricing set-up, providing confidence intervals for the locational marginal prices.

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