Journal of Mathematical Economics 21 (1992) 483–508. North-Holland

Optimal licensing of cost-reducing innovation*

Morton I. Kamien**

Northwestern University, Evanston IL, USA

Shmuel S. Oren

University of California, Berkeley CA, USA

Yair Tauman

State University of New York at Stony Brook, Stony Brook NY, USA and Tel Aviv University, Tel Aviv, Israel

Submitted August 1990, accepted July 1991

We analyze licensing of a cost reducing innovation to an oligopolistic industry, and extend previous work by Kamien and Tauman (1986) and Katz and Shapiro (1986) in two directions. First, our analysis applies to a wider class of demand functions than linear ones. Second, we derive a simple optimal licensing mechanism for the patentee. We also examine three licensing mechanisms commonly discussed in the literature and observed in practice. Auctioning of a fixed number of licenses is compared to a fixed license fee and to a per unit royalty in terms of the patentee's profit, licensees' profit, industry structure, and the product's price. The analysis is conducted in terms of a non-cooperative game involving the patentee and n identical firms. In this game the patentee acts as a Stackelberg leader selecting a licensing strategy by taking into account the reaction and competitive interaction of the firms. The competitive interaction among the firms is modeled explicitly, both as a quantity (Cournot) and as a price (Bertrand) subgame in a market for a homogeneous product. We examine the implications of the three licensing strategies and how they depend on the relative magnitude of the innovation, the number of firms, and the price elasticity of demand. Licensing by means of a royalty is inferior to the other modes, both for consumers and the patentee. The firms' profits decline under both the auction and the fixed fee policies relative to their pre-innovation profits. Finally, it is shown that auctioning licenses is the patentee's optimal strategy when the magnitude of innovation is not too small. However, this does not hold for an arbitrary innovation.

1. Introduction

The focus of this work is the licensing of an innovation that reduces the

Correspondence to: Prof. M. Kamien, M.E.D.S., Kellogg School of Management, Northwestern University, Evanston, IL 60201, USA.

*We thank Avraham Beja for bringing the work by Stobiecki (1975) to our attention. The referees added very useful suggestions for which we are grateful.

**Research was provided by the Heizer Entrepreneurship Research Center.

0304-4068/92/$05.00

production cost of an existing good or service in a competitive or oligopolistic market. We analyze and compare the implications for a licensor, licensees and consumers of three common licensing strategies for reaping the fruits of a patented invention. In particular we compare auctioning a fixed number of licenses to the highest bidders, selling them for a flat fee, or for a per unit royalty. Early work on licensing of cost reducing innovation can be traced back to Arrow (1962) who focused on the question of whether it is more profitable to innovate in a competitive or monopolistic industry. Kamien and Tauman (1984, 1986) analyzed alternative licensing strategies using a game theoretic formulation to account explicitly for the competitive interaction among potential licensees and the patentee's ability to exploit it. In the former analysis, the optimal fixed fee plus royalty licensing strategy was described, and in the latter, optimal fixed fee licensing alone was compared with licensing solely by means of a royalty. Both analyses are limited to the case of a linear demand function for the product to which the innovation applies. Katz and Shapiro (1986) studied licensing by means of an auction without explicitly modeling the underlying competitive interaction among potential licensees. Their general analysis does not disclose the effect of licensing strategies on market structure, firms' profits, and the market price. A survey of this literature is provided in Kamien (1990).

Our analysis follows Kamien and Tauman's (1986) approach of explicitly modeling the competitive interaction among licensed and unlicensed firms, from which their demand for licenses derives. We consider both Cournot and Bertrand type competition among them. The patentee is treated as a leader in a Stackelberg type game in which the potential licensees, the followers, compete in the product market. Thus, the patentee chooses a licensing mode to maximize his profit, taking into consideration the firms' anticipated reaction. The paper contains two main contributions. First, results obtained by Kamien and Tauman (1986) for the linear demand case are extended to a wider class of demand functions. Second, it is shown that a patentee who cannot observe or control individual firm's production can still extract the highest potential licensing profit of an innovation that is not too small by a simple auction.

Our analysis discloses that, if the firms are Cournot competitors and if the magnitude of innovation is not too small, the innovator optimally licenses a non-drastic innovation to $K=c/\varepsilon\eta(c)$ firms under both auction and fixed fee licensing, where c is the pre-innovation (fixed) marginal cost, ε is the magnitude of the innovation (i.e., the innovation reduces the marginal cost of production from c to $c-\varepsilon$) and $\eta(\cdot)$ is the price elasticity of demand. The number K, independent of the initial number of firms n, is the number of licensees for which the post-innovation market price equals the pre-innovation competitive price level c. Consequently, the unlicensed firms are driven from the industry and only K remain. On the other hand, a drastic innovation, is exclusively licensed to a single firm, that charges a monopoly price of c or less. If the cost reduction is relatively small it is best for the patentee to license all the industry's firms. It is also shown that, irrespective of the magnitude of innovation, all the firms are worse off and the market price declines below its pre-innovation level.

We find that licensing by means of linear royalties is inferior for both the patentee and consumers relative to the auction and the fixed fee licensing strategies. On the other hand, each firm's profit is at least as high as its pre-innovation level.

As for general licensing mechanisms, we consider all which do not depend on firms' production levels. Katz and Shapiro found that a patentee can achieve the maximum potential licensing profit by a two stage mechanism. In the second stage the patentee auctions k licenses if in the first stage all the firms in the industry pay a pre-determined entry fee E. Otherwise, if a non-empty set R of firms refuse to pay this fee, then the firms outside R all receive a refund E and a free license while the firms in R are not licensed. Now, if the patentee chooses E and k, then, in equilibrium, each firm pays the fee E, which, with the revenues from the second stage auction, yields the highest potential licensing profit.

The difficulty with this mechanism is that it relies on the patentee's binding commitment which is not credible (to provide licenses free of charge and realise zero profit even if only one firm refuses to pay the entry fee). We have resolved this difficulty by showing that a simple auction is an optimal mechanism for the patentee as long as the magnitude of the innovation ε, is not too small, namely, $\varepsilon \geqq 2c/(n\eta(c))$. As for relatively small $\varepsilon > 0$, an auction is not the patentee's optimal mechanism. For this case another two stage mechanism is proposed. In its first stage the patentee makes a particular licensing offer to the industry's firms. If the offer is not unanimously accepted the refusniks are punished and, in the second stage, the unpunished firms are licensed at a 'reduced' rate. In this mechanism, it is essentially a dominant strategy for every firm to accept the patentee's initial offer and the patentee achieves the highest potential licensing profit.

In the next section we state our models formally. Section 3 contains some results on Cournot oligopoly used in the subsequent analysis. Section 4 addresses the auction mechanism while in section 5 and 6 fixed fees and royalties are examined. Section 7 deals with the optimal licensing mechanism. In section 8 we reexamine the above licensing policies when firms are engaged in Bertrand competition rather than Cournot competition.

2. The model

We consider an industry consisting of $n \geqq 2$ identical firms producing the same good with a linear cost function $C(q) = cq$, where q is the quantity produced and $c>0$ is the constant marginal cost of production. The market for the good is characterized by Cournot competitors facing a downward sloping inverse demand function $P(q)$, where $P(0)>c$.

We assume that a cost reducing innovation developed by a patentee, who is not an industry member, can lower the marginal cost of production from c to $c-\varepsilon$, $\varepsilon>0$. Our first goal is to analyze the implications of three observable licensing policies:

(1) Auctioning a limited number, k, of licenses through a sealed bid English auction. The highest k bidders get licenses. Ties are resolved by the patentee.
(2) A flat pre-determined license fee α at which any firm that wishes to can purchase a license (as part of his strategy).
(3) A fixed royalty payment, r, per unit of production.

The interaction between the patentee, the n firms, and their market is characterized by the following three stage game. In the first stage, the patentee announces a licensing policy along either with the corresponding prices, royalty rate, or number of licenses to be auctioned. In the second stage, the firms simultaneously and independently decide whether or not to purchase a license or how much to bid, and their decisions determine the set of licensees. In the third stage, the set of licensees becomes common knowledge and all n firms simultaneously and independently determine their production levels. The patentee's payoff is the total licensing profit, while the firms' payoffs are their profits net of license expenses. The specific forms of these payoff functions depend on the licensing policy and will be specified in the following sections.

In analyzing the game described above we restrict ourselves to the subgame perfect equilibrium concept, meaning that it is a Nash equilibrium which induces a Nash equilibrium in every subgame. The meaning of this restriction in the present context is that in the game's third stage the licensees and non-licensees compete as a Cournot oligopoly under every possible outcome of the second stage. The Cournot equilibria corresponding to these subgames are taken into account by the firms in determining their decisions in the second stage. This in turn is taken into consideration by the patentee in determining his optimal licensing strategy.

The following assumptions on the demand function are used throughout.

Assumption 1. The total revenue function, $qP(q)$, is strictly concave in q.

Assumption 2. The demand function, $Q(p)$, is decreasing, differentiable for $p>0$ and the price elasticity $\eta(p)=-pQ'/Q$ (where $Q'=\mathrm{d}Q/\mathrm{d}p$) is a non-decreasing function of p.

3. General properties of the Cournot oligopoly subgame

Before analyzing specific licensing policies, we list some properties related to Cournot equilibria relevant to this paper. Most of them are well known or easy to derive.

Consider a Cournot oligopoly consisting of a set $\boldsymbol{n}$ of firms engaged in the production of the same good. The set $\boldsymbol{n}$ is partitioned into two subsets, $\boldsymbol{s}$ and $\boldsymbol{n}\backslash\boldsymbol{s}$, corresponding to the two possible production technologies, with production costs $c-\varepsilon$ and c. Suppose that the sets $\boldsymbol{s}$ and $\boldsymbol{n}\backslash\boldsymbol{s}$ consists of k and $n-k$ firms, respectively. Then, the Cournot equilibrium possesses the following properties:

(i) The equilibrium market price p satisfies

$$1-1/(n\eta(p))=(c-\varepsilon k/n)/p \quad \text{for} \quad k\leqq K=c/(\varepsilon\eta(c)), \tag{1a}$$

and

$$1-1/(k\eta(p))=(c-\varepsilon)/p \quad \text{for} \quad k\geqq K. \tag{1b}$$

Since $\eta(p)$ is non-decreasing in p, eqs. (1a) and (1b) uniquely determine, for any k, $0\leqq k\leqq n$, an equilibrium price $p=p(k)$. Notice that $p(K)=c$ and for $k>K$, $p(k)<c$.

(ii) The equilibrium production levels for $k\leqq K$ are:

$$q^{s}=\frac{Q(p)[c-\varepsilon+\varepsilon(n-k)\eta(p)]}{nc-k\varepsilon}, \tag{2a}$$

$$q^{n\backslash s}=\frac{Q(p)[c-\varepsilon k\eta(p)]}{nc-k\varepsilon}, \tag{2b}$$

where $p=p(k)$, and $Q(p)$ is the total quantity demanded at the price p. For $k\geqq K$,

$$q^{s}=\frac{Q(p)}{k} \quad \text{and} \quad q^{n\backslash s}=0. \tag{2c}$$

(iii) For $k\leqq K$, the equilibrium profits $\Pi^{s}(k)$ and $\Pi^{n\backslash s}(k)$ of each firm in $\boldsymbol{s}$ and $\boldsymbol{n}\backslash\boldsymbol{s}$, respectively, are:

$$\Pi^{s}(k)=\frac{(q^{s})^{2}}{\eta(p)}\frac{p}{Q(p)}=-\frac{(p-c+\varepsilon)^{2}}{P'}, \tag{3a}$$

and

$$\Pi^{n\setminus s}(k)=\frac{(q^{n\setminus s})^2}{\eta(p)}\frac{p}{Q(p)}=-\frac{(p-c)^2}{P'}, \tag{3b}$$

where $p=p(k)$ and $P'=\mathrm{d}P/\mathrm{d}Q$.

Lemma 1. *(i) The Cournot equilibrium price* $p=p(k)$ *decreases in* k *for* $1\leqq k\leqq n$.
(ii) For each k, $0\leqq k\leqq K$, $np+P'Q=nc-k\varepsilon$.
(iii) For each k, $0\leqq k\leqq K$, $Q\,\partial P'/\partial k+(n+1)\,\partial p/\partial k=-\varepsilon$.

Proof. See appendix.

Throughout the paper the number of licensees, k, is treated as a continuous variable to determine if certain relevant functions of k are increasing or decreasing. However, this does not have any effect on the results.

Lemma 2. *The profit function* $\Pi^{n\setminus s}(k)$ *of a nonlicensee decreases with* k *for* $k<K$ *and* $\Pi^{n\setminus s}(k)=0$ *for* $k\geqq K$. *The profit function* $\Pi^{s}(k)$ *of every licensee decreases with* k.

The proof of this lemma follows from Assumption 1, the general properties of the Cournot subgame, and Lemma 1.

In our subsequent analysis we use the concept of drastic innovation. Following Arrow (1962), a cost reducing innovation is *drastic* iff the monopoly price under the new technology does not exceed the competitive price under the old technology.

Lemma 3. *Let* c *be the fixed marginal cost under the old technology and let* $c-\varepsilon$ *be the reduced marginal cost under the new technology. Then, an innovation is drastic iff* $\varepsilon\geqq c/\eta(c)$.

The proof follows by Assumption 2 and (1b).

4. The auction policy

Let us define the payoff functions for the game resulting from the auction policy. Assuming that k licenses are auctioned, denote by $\boldsymbol{b}=(b_1(k),\dots,b_n(k))$ the n bids submitted by the n firms, respectively. Let $s=s(k)$ be the set of the k licensees and denote by $\boldsymbol{q}=(q_1(\boldsymbol{b},s),\dots,q_n(\boldsymbol{b},s))$ the respective production levels of the n firms. Then the payoff functions of the patentee Π_0^{a} (where the superscript, a, stands for auction), and those of the firms $\Pi_1^{\mathrm{a}},\dots,\Pi_n^{\mathrm{a}}$ are defined as follows:

$$\Pi_0^{a}=\Pi_0(\boldsymbol{b},s\boldsymbol{q})=\sum_{i\in s} b_i(k),$$

$$\Pi_i^{a}=\begin{cases}(p-c+\varepsilon)q_i-b_i & \text{for} \quad i\in S\\ (p-c)q_i & \text{for} \quad i\notin S,\end{cases} \tag{4}$$

where

$$p=P\left(\sum_{i=1}^{n} q_i\right).$$

First we analyze the case of a non-drastic innovation.

Proposition 1. Consider a non-drastic innovation where $\varepsilon\geqq 2c/((n+1)\eta(c))$. *Then:*

(*i*) *The equilibrium number of licensees is given by* $K=c/(\varepsilon\eta(c))$.
(*ii*) *The post-innovation market price is c, i.e., the competitive price under the old technology.*
(*iii*) *Each unlicensed firm drops out of the market, while each licensee produces* $Q(c)/K$ *units but yields all his profit to the patentee.*
(*iv*) *The patentee's profit is* $\Pi_0^{a}=\varepsilon Q(c)$.

The inequality $\varepsilon\geqq 2c/((n+1)\eta(c))$ cannot be improved. It can be shown that Proposition 1 does not hold for linear demand if $\varepsilon<2c/((n+1)\eta(c))$ [see Kamien and Tauman (1986)].

If K is not an integer then the number, K^*, of licensees is either $\lceil K\rceil$ or $\lceil K\rceil+1$ and the market price, $p(k^*)$, will be either slightly above or below c (depending on whether $k^*=\lceil K\rceil$ or $k^*=\lceil K\rceil+1$, respectively). If $p(k^*)>c$ then unlicensed firms will not be driven from the industry, but their share of industry wide profits will be minor (otherwise, the patentee will sell $\lceil K\rceil+1$ licenses and drive the market price below c).

Proof. Suppose that the patentee auctions k licenses, $1\leqq k\leqq n-1$. Then[1] each firm's willingness to pay for a license is given by $\Pi^{s}(k)-\Pi^{n\setminus s}(k)$, where $\Pi^{s}(k)$ and $\Pi^{n\setminus s}(k)$ are the Cournot equilibrium profits of licensees and non-licensees, respectively. Notice that $\Pi^{n\setminus s}(k)$ is a licensee's opportunity cost when k licenses are auctioned. Hence, the patentee's equilibrium profits are

[1]If n licenses are auctioned, each firm is assured a license and will bid as little as possible. In order to induce firms to bid their reservation price, the patentee must limit the miniumum bid to that price. Thus, for $k=n$, the auction policy is equivalent to the fixed fee policy discussed in the next section.

$$\Pi_0^{\mathrm{a}} = \max_{1 \leqq k \leqq n-1} k[\Pi^s(k) - \Pi^{n\setminus s}(k)]. \tag{5}$$

By (3a) and (3b) it can be verified that

$$\Pi_0^{\mathrm{a}}(k) = k[\Pi^s(k) - \Pi^{n\setminus s}(k)]$$
$$= \begin{cases} \dfrac{\varepsilon k Q(p)}{nc - k\varepsilon}[2c + \varepsilon(n\eta - 2k\eta - 1)], & 1 \leqq k \leqq K, \\ (p - c + \varepsilon)Q(p), & k \geqq K. \end{cases} \tag{6}$$

To complete the proof of part (i) of the proposition, we use the following lemma.

Lemma 4. (*i*) *For each* $k \geqq K$, *the function* $\Pi_0^{\mathrm{a}}(k)$ decreases in k.
(*ii*) *For* $k \leqq K$, *and* $\varepsilon \geqq 2c/((n+1)\eta(c))$, *the patentee's revenue* Π_0^{a} *increases in* k.

Proof. See appendix.

It follows by Lemma 4 that $\Pi_0^{\mathrm{a}}(k)$ attains its maximum at $k = K$. This completes the proof of Part (i) of Proposition 1.

Part (ii) of the proposition follows by substituting $k = K$ in (1a).

Part (iii) follows from (2c) according to which $q^{n\setminus s} = 0$ for $k \geqq K$.

Part (iv) follows from (6) by substituting $p = c$ and $k = K$. □

The implication of Proposition 1 is that for a non-drastic but sufficiently important innovation the patentee induces an oligopoly consisting of fewer firms than originally, all of whom, however, employ the new technology. This number of licensees depends on the relative magnitude of the innovation, and the demand elasticity at the competitive price c under the old technology, but not on the original number of firms n. Furthermore, as every firm's profits drop to zero, each is worse off, whether it is a licensee or not, relative to its pre-innovative profits. Consumers, on the other hand, are better off as the market price falls to c. The patentee manages to extract the licensee's total operating profits, $\varepsilon Q(c)$. This profit equals what he could realize in Bertrand competition by driving the price to c and becoming the industry's sole producer. Further discussion of the Bertrand model is provided in section 7.

Next we deal with innovations of relatively small magnitude.

Proposition 2. There exists an $\bar{\varepsilon}, 0 < \bar{\varepsilon} \leqq c$, *such that for any* $\varepsilon, 0 \leqq \varepsilon \leqq \bar{\varepsilon}$, *each firm, except perhaps one, becomes a licensee. Namely,* $k^* \geqq n - 1$.

The proof appears in the appendix. The result obtains for ε's which are too small to drive the market price below c and to force unlicensed firms to exit the industry. That is, ε should be sufficiently small to ensure $n \leqq K$. In the linear demand case $k^* = n$ for $\varepsilon \leqq 2c/(3n-1)\eta(c)$ [see Kamien and Tauman (1986)].

We proceed to properties of the non-drastic innovation case which hold in general.

Proposition 3. Consider a non-drastic cost reducing innovation. Then:

(*i*) *The equilibrium number of licensees does not exceed $c/(\varepsilon\eta(c))$.*
(*ii*) *Every frim is worse off relative to its pre-innovation profit.*
(*iii*) *The innovation results in a lower market price.*

Proof. The first two parts can be found in Katz and Shapiro (1986). The third part follows from Lemma 1 (part (i)).

We now proceed to the case of a drastic innovation under an auction policy. By definition, the innovation is drastic iff the monopoly price under the new technology falls below c. In this case, if only one license is auctioned ($k=1$), the Cournot equilibrium price, given by (1), falls below c and the exclusive licensee becomes a monopolist but the patentee realizes the entire monopoly profit. This establishes the following proposition.

Proposition 4. The industry is monopolized if and only if the innovation is drastic. In this case the patentee extracts the entire monopoly profit, the price falls below c, and the single licensee makes zero profit.

5. Fixed fee policy

In this section we discuss the implications of a fixed fee policy where the patentee sets a uniform license fee α, and does not restrict the number of licensees. The strategy of firm i is characterized by a pair (τ_i, q_i) where $\tau_i(\alpha)$ is a function from E^2_+ to $\{0,1\}$ with the convention that $\tau_i(\alpha)=1$ iff i purchases the license and $\tau_i(\alpha)=0$, otherwise. The second element, q_i, determines for each α and each subset s of licensees the ith firm's production level, $q_i = q_i(\alpha, s)$. Its payoff functions are defined by

$$\Pi_i(\alpha,(\tau_1,q_1),\ldots,(\tau_n,q_n)) = \begin{cases} (p-c+\varepsilon)q_i - \alpha_i & \text{for} \quad i \in s, \\ (p-c)q_i & \text{for} \quad i \notin s, \end{cases} \tag{7}$$

where $p = (\sum_{i=1}^n q_i)$. The patentee's profit, Π^{f}_0 (the superscript f stands for fee), is

$$\Pi_0^{\mathrm{f}}(\alpha, (\tau_1, q_1), \ldots, (\tau_n, q_n)) = \alpha k,$$

where k is the number of firms in s.

The essential difference between the fee and the auction policies stems from the firms's opportunity cost of having a license when there are k licensees under a fee policy being $\Pi^{n\setminus s}(k-1)$, as opposed to $\Pi^{n\setminus s}(k)$ in the auction case. The reason for this is that, under the fee policy, a licensee's decision to relinquish a license reduces the number of licensees by one, while in the auction case the number of licensees is pre-determined. Consequently, the amount a firm is willing to pay to be one of k licensees under the fee policy, is given by

$$w(k) = \Pi^s(k) - \Pi^{n\setminus s}(k-1).$$

Let α be the fee charged by the patentee and let s be a set of k licensees. A buyer in s will not deviate from his decision iff $\alpha \leqq w(k)$. A non-buyer will not deviate from his decision iff $\alpha \geqq w(k+1)$. Consequently, it is required that $w(k+1) \leqq \alpha \leqq w(k)$ and hence, the number k of licensees is optimally supported by a fee $\alpha^* = w(k)$ as long as $w(k) \geqq w(k+1)$. Therefore, the patentee's optimal profit level Π_0^{f} under the fixed fee policy is given by

$$\Pi_0^{\mathrm{f}} = \max_k kw(k) \tag{8}$$

over all k such that $w(k) \geqq w(k-1)$, $0 \leqq k \leqq n$.

The existence of an optimal solution to (8) can be easily established [similarly to the proof of Proposition 2 of Kamien et al. (1988)]. Hence, the game resulting from the fixed fee policy has a subgame perfect equilibrium in pure strategies.

One of the complications that may arise in a fixed fee policy is that the fee α does not necessarily induce a unique number, k, of licensees. As indicated above, for any given α, the equilibrium number k of licensees must satisfy

$$\alpha = \Pi^s(k) - \Pi^{n\setminus s}(k-1) = w(k).$$

In general, this equation may have multiple solutions for k, as shown in fig. 1. In this figure we have three values of k satisfying $\bar{\alpha} = w(k)$. Two of them, k_1 and k_3, can be obtained in equilibrium since $w(\cdot)$ decreases at both points. A licensee's profit, net of the license fee at any equilibrium point equals his opportunity cost. Hence, the licensee's profits corresponding to k_1 and k_3 are $\Pi^{n\setminus s}(k_1 - 1)$ and $\Pi^{n\setminus s}(k_3^{-1})$. Since $\Pi^{n\setminus s}(k)$ decreases in k (Lemma 2), both licensed and unlicensed firms are better off at the equilibrium point k_1 or, in general, at the equilibrium point with the smallest number of licensees corresponding to a given fee α. On the other hand, the patentee will obviously obtain the highest profit $(k\alpha)$ at the equilibrium point with the

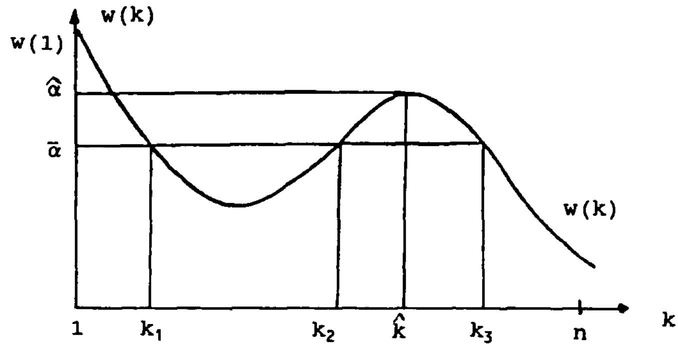

Fig. 1. Willingness to pay for a license as a function of the number of licensees, under a fixed fee policy.

highest k, for the given α (in our case, k_3). Since the patentee only controls α he cannot ensure the k he desires. Thus, even though he might have determined a fee α expecting to obtain k_3 licensees, the equilibrium may result with only k_1 licensees. Notice that for the case depicted in fig. 1, if $w(1)$ were lower than $\hat{\alpha}$, then k_1 would not be an equilibrium outcome since by increasing the fee from $\bar{\alpha}$ to $\hat{\alpha}$ the patentee 'could force' the equilibrium to move to $\hat{k}$ and increase his profits. It should also be noticed that any point, such as k_2, at which $w(\cdot)$ increases, cannot be an equilibrium outcome since any additional non-licensee, at such a point, would be willing to purchase the license at a price higher than $\bar{\alpha}$.

The phenomenon of multiple equilibria described above will not occur if $w(k)$ decreases in k, as is the case when the demand function is linear [see Kamien and Tauman (1986)]. But even if $w(k)$ is not decreasing in k, multiple equilibria of the above type will not occur if the fixed fee policy is extended to allow discriminatory fees. That is, the patentee is allowed to set a different α for each firm. Notice that any equilibrium of this game can be supported by a *uniform* (*nondiscriminatory*) *fee*, but the fact that the patentee can potentially increase his profit by offering licenses for a small fee to an additional k_3-k_1 firms will eliminate k_1 as a possible equilibrium outcome. In general, any equilibrium of a discriminatory fee policy game can be supported by a uniform fee α which uniquely determines the number k of licenses. This number is the largest k at which $w(k)$ decreases and $w(k)=\alpha$. [A similar phenomenon in the context of priority pricing was analyzed by Stobiecki (1975)].

Allowing the use of different fees for different firms we have the following proposition.

Proposition 5. Under the fixed fee policy (*with possibly discriminatory fees*):

(*i*) *The equilibrium number of licensees is uniquely determined and it is bounded by* $K+1$. *Furthermore, it can be supported by a uniform fee.*
(*ii*) *Each firm is worse off relative to its pre-innovation profit level (with the exception of an exclusive licensee who retains his pre-innovation profit level).*
(*iii*) *The patentee's revenue is strictly lower under an optimal fixed fee than under an optimal auction.*

Proof. See appendix.

It should be mentioned that while the patentee prefers an auction to fixed fee licensing, consumers may prefer the opposite. For the case of linear demand, the market price under a fee does not exceed the market price under auction. If the magnitude ε of the innovation is not 'too' large, then the price under a fee is strictly lower than under an auction.

6. The royalty policy

In this section we discuss the implication of the patentee charging each licensee a uniform per-unit of production royalty r. After r is announced firms decide independently and simultaneously whether to pay it or continue to produce with the old technology. Firm i's strategy is a pair (τ_i, q_i) where τ_i is a function from E^1 to $\{0,1\}$ with the convention that $\tau_i(r)=1$ if firm i purchases the license and $\tau_i(r)=0$, otherwise. The second component q_i determines for each royalty r and each subset s of licenses the ith firm's production level $q_i=q_i(r,s)$. Its payoff functions are determined as follows:

$$\Pi_i(r,(\tau_1,q_1),\ldots,(\tau_n,q_n))=\begin{cases}(p-c+\varepsilon-r)q_i, & i\in s,\\ (p-c)q_i, & i\notin s,\end{cases}$$

where $p=P(\sum_{i=1}^n q_i)$. The patentee's profit, Π_0^r, is defined by

$$\Pi_0^r=\Pi_0^r(r,(\tau_1,q_1),\ldots,(\tau_n,q_n))=r\sum_{i\in s} q_i.$$

Notice first that for $r<\varepsilon$ every firm becomes a licensee since, free of charge, it lowers its marginal cost from c to $c-\varepsilon+r$. Next observe that $r=\varepsilon$ implies that both a licensed and an unlicensed firm will produce with the same marginal cost, c. Hence, for $r=\varepsilon$, a firm is indifferent between purchasing and not purchasing a license. Nevertheless, in a subgame perfect equilibrium with $r=\varepsilon$ we must have $k^*=n$ (otherwise, the patentee will be better off by slightly reducing his royalty below ε to ensure $k^*=n$). Finally, notice that if $r>\varepsilon$ no firm will be a licensee. Consequently, in any subgame perfect equilibrium of this game every firm becomes a licensee.

The first-order condition of the firm's profit maximization is

$$p+P'q_1=c-\varepsilon+r, \qquad i\in \mathbf{n}.$$

Thus, the equilibrium price is

$$p=c-\varepsilon+r-QP'/n. \tag{9}$$

Proposition 6. Under the optimal linear royalty policy:

(i) Each firm becomes a licensee.
(ii) The market price strictly exceeds the market price under the auction or fixed fee licensing.
(iii) The patentee's revenues are lower than with a license auction.
(iv) Suppose that $\eta(c)<1$. Then for sufficiently large n each firm's profit is at least as high as its pre-innovation profit.

Proof. See appendix.

Proposition 6 asserts that for both the patentee and consumers a uniform royalty is inferior to an auction. On the other hand, in a sufficiently competitive industry where $\eta(c)<1$, the firms are all better off under the royalty policy. Notice, that if $Q=a-P$, then $\eta(c)<1$ iff $a>2c$.

7. Optimal licensing mechanism

In this section we analyze an optimal licensing mechanism of a patentee who cannot control or effectively observe firms' production levels. Thus, a feasible mechanism cannot be contingent on firms' production levels and therefore a linear royalty, for instance, is not feasible. It is assumed that the firms, after the licensing process is completed, engage in Cournot competition. Denote by G the class of all feasible licensing mechanisms for the patentee. The class G contains more than the auction or the fixed fee mechanisms. It may contain a variety of sequential mechanisms. For example, consider an industry of $n=3$ firms and let $K=c/(\varepsilon\eta(c))=2$. [Recall that K is the smallest k such that $p(k)=c$.] Let $\delta>0$ be a small number and let M_δ be the following sequential mechanism. The patentee first approaches firm 1. He offers it an exclusive license for the price $\alpha_1=\Pi^s(1)-\delta$. If it refuses to purchase the license, then firm 2 is approached and offered a (non-exclusive) license for the price $\alpha_2=\Pi^s(2)-\Pi^{n\setminus s}(1)-\delta$. Then firm 3 is approached and offered a license for a price which depends on firm 2's decision. If firm 2 purchases a license, then the price to firm 3 is the same as to firm 2, namely, α_2. Otherwise, it is $\alpha_3 = \Pi^s(1) - \Pi^{n\setminus s}(0) - \delta$. If this mechanism is common knowledge among the firms then the unique backward induction outcome is that firm 1 becomes the exclusive licensee. Indeed, suppose that firm 3 is approached by the patentee. If firm 2 refused to buy, then firm 3's net profit is $\Pi^{n\setminus s}(0) + \delta$ if it purchases a license and $\Pi^{n\setminus s}(0)$ if it does not. Hence, firm 3 will purchase a license. If firm 2 purchased a license then firm 3's net profit is $\Pi^{n\setminus s}(1) + \delta$ if it purchases a license and $\Pi^{n\setminus s}(1)$ if it does not. Hence, in both cases, firm 3 is better off purchasing a license. Firm 2, if approached by the patentee, takes into account that whatever its decision, firm 3 will purchase a license. Thus, if firm 2 purchases a license for the price α_2, its net profit is $\Pi^{n\setminus s}(1) + \delta$ while if it does not its profit is $\Pi^{n\setminus s}(1)$. Therefore, if firm 2 is approached by the patentee, then both firms 2 and 3 will purchase licenses for the price α_2. Taking this into account, firm 1 knows that if it pays the price $\alpha_1 = \Pi^s(1) - \delta$ it will be the exclusive with a net profit δ. Otherwise, it will be the only unlicensed firm with profit $\Pi^{n\setminus s}(2)$. Since $K = 2$, $p(2) = c$, and thus $\Pi^{n\setminus s}(2) = 0$. Consequently, firm 1 will purchase the license and be the exclusive licensee.

The patentee's profit under the mechanism M_δ is $\Pi^s(1) - \delta$ while for an exclusive license he can obtain, with an auction, $\Pi^s(1) - \Pi^{n\setminus s}(1)$. If the innovation is not drastic, then $\Pi^{n\setminus s}(1) > 0$. Therefore, an auction for an exclusive license yields the patentee lower profit than M_δ does. Sequential licensing mechanisms of a similar nature are discussed in Tauman and Weiss (1990) in a different context. Their model deals with a monopolistic industry where barriers to entry are due to a large fixed cost. The innovation reduces the fixed cost and entry becomes profitable. This is an asymmetrical model where the incumbent's willingness to pay is different from that of a potential entrant.

Our next goal is to show that if ε, the magnitude of innovation is not 'too small', then the auction mechanism (described in section 4) is optimal for the patentee as it maximizes his total rents over all possible licensing mechanisms in G. This does not contradict our previous statement about M_δ since it was compared with the auction of an exclusive license only. It is known from Proposition 1 that the optimal number of licenses to be auctioned is K, which is 2 in the example above. Also, we will show that for a relatively small ε, an auction is not in general the patentee's best strategy.

Let $\boldsymbol{s} \subseteq \boldsymbol{n}$, $\boldsymbol{n} = \{1, 2, \ldots, n\}$ and let $\chi(\boldsymbol{s}) \in R^n_+$ be the Cournot equilibrium profit vector corresponding to an industry with a set of $\boldsymbol{n}$ firms and a subset $\boldsymbol{s}$ of licensees. Let $\boldsymbol{s}^*$ be a subset of $\boldsymbol{n}$ which maximizes total industry profits. That is,

$$\boldsymbol{s} \in \arg\max_{\boldsymbol{s} \subseteq \boldsymbol{n}} \sum_{i \in \boldsymbol{n}} \chi_i(\boldsymbol{s}). \tag{10}$$

Denote $\chi^* = \chi(s^*)$, i.e., χ^* is the Cournot equilibrium profit vector with the highest total industry profits. The lowest profit level of firm $i \in \mathbf{n}$ is attained for $s = \mathbf{n} \backslash \{i\}$, that is, when every other firm produces with the new technology. Denote this lowest profit level Π_1, i.e., $\Pi_1 = \chi_i(\mathbf{n} \backslash \{i\})$. Finally, let μ be the Cournot equilibrium profit level prior to the innovation. With the above notation, $\mu = \chi_i(\phi)$.

Proposition 7. The patentee's potential licensing profit is bounded above by $\sum_{i \in \mathbf{n}} \chi_i^* - n\Pi_1$.

Proof. Obviously, the most the patentee can extract is the highest total industry profit which is $\sum_{i \in \mathbf{n}} \chi_i^*$. Since each firm can guarantee itself at least Π_1 (by just producing with the old technology) he can at most extract $\sum_{i \in \mathbf{n}} \chi_i^* - n\Pi_1$.

Proposition 8. If $\varepsilon \geqq 2c/(n\eta(c))$, *then the upper bound* $\sum_{i \in \mathbf{n}} \chi_i^* - n\Pi_1$ *of the patentee's potential licensing profit is achieved by an auction. The optimal number of licensees is* $K = c/(\varepsilon\eta(c))$ *and every unlicensed firm is driven out of the industry.*

Proof. If the innovation is drastic ($K \leqq 1$), then the patentee extracts the entire monopoly profit (under the auction policy) (Proposition 4) and the upper bound is clearly achieved. Suppose next that the innovation is not drastic. Let

$$H(k) = k\Pi^s(k) + (n-k)\Pi^{\mathbf{n}\backslash s}(k).$$

The highest total industry profit is $\max_k H(k)$. Our purpose is to show that $H(k)$ is maximized for $k = K$. This will imply that the patentee's profit is bounded from above by $H(K)$. But $H(K) = K\Pi^s(K) = \varepsilon Q(c)$, and this level is achieved by the auction policy when $\varepsilon \geqq 2c/(n\eta(c))$. By Lemmas 2 and 4, H decreases for $k \geqq K$. To complete the proof it is sufficient to show that $H(k) \leqq \varepsilon Q(c)$ for any $k \leqq K$.

$$H(k) = \frac{k(p-c+\varepsilon)^2 + (n-k)(p-c)^2}{-P'} = \frac{k\varepsilon^2 + 2k\varepsilon(p-c) + n(p-c)^2]Q}{-P'Q}.$$

By Lemma 1

$$H(k) = \frac{[k\varepsilon^2 + 2k\varepsilon(p-c) + n(p-c)^2]Q}{np - nc + k\varepsilon}.$$

Since $p \geqq c$ $H(k) < \varepsilon Q(c)$ if

$$(*)\ [2k\varepsilon + n(p-c)]Q \leqq n\varepsilon Q(c).$$

Let

$$L = [2k\varepsilon + n(p-c)]Q. \tag{11}$$

By (1a)

$$k\varepsilon = \frac{p}{\eta(p)} - n(p-c).$$

This together with (11) imply that

$$L = \left[\frac{2p}{\eta(p)} - n(p-c)\right]Q \leqq \frac{2p}{\eta(p)}Q(p). \tag{12}$$

Let us show that $(p/(\eta(p))Q(p)$ is decreasing in p. By the definition of η this expression is $Q^2/(-Q')$. Since $P'(q)<0$ it is sufficient to prove that $q^2(-P'(q))$ is increasing in q. But $\partial[-q^2P'(q)]\partial q = -q(2P' + qP'')$. Hence $q^2(-P'(q))$ is increasing in q iff $2P' + qP'' < 0$. But the last inequality follows by the strict concavity of $qP(q)$ (Assumption 1). Therefore, since $p \geqq c$ we obtain by (12)

$$L \leqq \frac{2p}{\eta(p)}Q(p) \leqq \frac{2c}{\eta(c)}Q(c).$$

By ($*$), it is left to show that $(2c/\eta(c))Q(c) \leqq n\varepsilon Q(c)$. But this is equivalent to $\varepsilon \geqq 2c/(n\eta(c))$. □

Next we proceed to describe a licensing mechanism M^* that guarantees the patentee the upper bond profit $\sum_{i \in \boldsymbol{n}} \chi_i^* - n\Pi_1$, for any magnitude ε of the innovation. The idea of this mechanism stems from Theorem 7.2 of Kamien et al. (1990). The *mechanism* M^* has two stages. First, the patentee selects a subset $\boldsymbol{s}^*$ satisfying (10). The firm in $\boldsymbol{s}^*$ are the potential licensees. Second, he offers each firm $i \in \boldsymbol{n}$ the option of paying a fee of $\chi_i^* - (\Pi_1 + \delta/n)$, where $\delta > 0$ is an arbitrary small number. Notice that this initial offer is made to any firm inside or outside $\boldsymbol{s}$. If all the firms agree to pay their fee then we say that the offer is accepted. In this case only, the firms in $\boldsymbol{s}^*$ obtain licenses and the industry engages in a Cournot oligopoly game. (Obviously, each firm's Cournot equilibrium net profit is $\Pi_1 + \delta/n$.) If, however, there is a non-empty subset $\boldsymbol{r}$ of firms refusing to pay the fee, we say that the offer is rejected. In this case, the firms in $\boldsymbol{n}\backslash\boldsymbol{r}$ (who accepted the offer) are offered licenses for the price α given by

$$\alpha=\chi_i(\boldsymbol{n}\backslash\boldsymbol{r})-\mu-\delta/n \quad \text{for} \quad i\in\boldsymbol{n}\backslash\boldsymbol{r}, \tag{13}$$

where $\mu=\chi_i(\emptyset)$ is each firm's pre-innovation profit. The firms in $\boldsymbol{r}$ are not entitled to purchase licenses. Notice that each licensee's net profit depends on the number of firms in $\boldsymbol{n}\backslash\boldsymbol{r}$ that purchase licenses for the price α. In any event, this profit is never below $\mu+\delta/n$.

Proposition 9. By eliminating dominated strategies it is a dominant strategy for each firm in $\boldsymbol{n}$ *to accept the initial offer.*

Proof. Let $i\in\boldsymbol{n}$. We will show that, independent of the actions taken by other firms (as long as they do not use dominated strategies), it is best for firm i to accept the patentee's initial offer. Since this is true for every $i\in\boldsymbol{n}$, all players must accept the initial offer. To prove this claim, we consider two cases. In the first case, some firms, other than i, reject the initial offer. In the second case, all other firms accept the initial offer. It will be sufficient to show that, in both cases, it is best for i to accept the initial offer. Let $\boldsymbol{r}\subseteq\boldsymbol{n}\backslash\{i\}$ be a subset of firms that reject the initial offer. Suppose first that $\boldsymbol{r}\neq\emptyset$. Let us compare i's payoff if he accepts or rejects the initial offer. If i accepts the initial offer, purchases a license and pays α, his net profit is $\chi_i(t)-\alpha$, where $\boldsymbol{t}\subseteq\boldsymbol{n}\backslash\boldsymbol{r}$ is the set of licensees (who pay the price α). If i does not purchase a license his profit is $\chi_j(\boldsymbol{t}\backslash\{i\})$ for $j\notin\boldsymbol{t}\backslash\{i\}$. By Lemma 2 and (13) we have

$$\chi_i(\boldsymbol{t})-\alpha\geqq\mu+\delta/n>\chi_j(\boldsymbol{t}\backslash\{i\}), \quad j\notin\boldsymbol{t}\backslash\{i\}. \tag{14}$$

That is, if $\boldsymbol{r}\neq\emptyset$ and i accepts the initial offer, then, regardless of the other firms' actions, i should purchase a license for the price α. Now, if $\boldsymbol{r}\neq\emptyset$ and if i rejects the initial offer, then for some $\boldsymbol{t}\subseteq\boldsymbol{n}\backslash\boldsymbol{r}$ his profits is $\chi_j(\boldsymbol{t})$, for $j\notin\boldsymbol{t}$, which by Lemma 2 is below $\mu+\delta/n$. Consequently, due to (14) we have that, if $\boldsymbol{r}\neq\emptyset$, then, regardless of the other firm's actions, it is best for i to accept the initial offer and purchase a license at the price α. It remains to check that i should accept the initial offer also if $\boldsymbol{r}=\emptyset$ – that is, if each firm, except i, accepts the initial offer. Suppose that i rejects the initial offer while everyone else accepts it. Then, by the previous case, applied to any j, $j\neq i$, it is dominant for j to purchase a license. So, we can delete j's strategies which lead him not to purchase a license in the case where i rejects the initial offer. Thus, the only undominated actions of all firms, other than i is to pay the fee. Hence, i will obtain $\chi_i(\boldsymbol{n}\backslash\{i\})=\Pi_1$ if he refuses the offer and $\Pi_1+\delta/n$ if he accepts it. Consequently, i should accept the initial offer regardless of the other firms' decisions.

Corollary. *For any $\delta > 0$, the mechanism M^* ensures the patentee a profit*

$$\sum_{i \in n} \chi_i^* - n\Pi_1 - \delta.$$

Finally, let us briefly examine the linear demand case.

Example. Consider a Cournot oligopoly ($n \geqq 2$) with a linear demand function

$$Q = a - p.$$

Then it can be verified that $\eta(p) = p/(a-p)$ and $K = c/(\varepsilon\eta(c)) = (a-c)/\varepsilon$. Now,
(i) if $\varepsilon \geqq 2(a-c)/(n+1)$, then the auction mechanism is optimal, the number of licensees is $K = (a-c)/\varepsilon$, the market price is c, and any unlicensed firm is driven out of the industry.
(ii) If $(a-c)/n < \varepsilon < 2(a-c)/(n+1)$ then M^* yields the patentee higher profits than an auction. Under M^* the optimal number of licensees k^* satisfies $k^* < (a-c)/\varepsilon$ and the market price exceeds c. In this case unlicensed firms continue to produce. However, each firm in $\boldsymbol{n}$ pays its entire profit to the patentee.
(iii) If $\varepsilon \leqq (1-c)/n$ then again M^* is preferred by the patentee to an auction and the optimal number of licensees, $\bar{k}$, is given by $\bar{k} = \min(k^*, n)$, where k^* is defined in (ii). This is the only case where the patentee cannot extract the entire industry profit, since

$$\Pi_1 = \left[\frac{a - c - (n-1)\varepsilon}{n+1}\right]^2 > 0.$$

8. Optimal licensing under Bertrand competition

The above analysis was directed to firms engaged in quantity (Cournot) competition. The analysis becomes much simpler if firms engage in price (Bertrand) competition. Price competition reduces each firm's profit to zero unless the new technology (with cost $c - \varepsilon$) is licensed exclusively to one firm. Thus, the patentee's profit is bounded from above by the highest total profit of firms engaged in Bertrand competition, which is obtained when the innovation is licensed to a single firm. In the case of a non-drastic innovation, the exclusive licensee sets a price equal to c and drives his competitors out of the market. Hence, $\Pi_1 = 0$ and the patentee's profit is $\varepsilon Q(c)$. In the case of a drastic innovation, the exclusive licensee sets the monopoly price, which is below c, and pays out the entire monopoly profit to the patentee. Thus, for a drastic innovation or a non-drastic innovation with $\varepsilon \geqq 2c/[(n+1)\eta(c)]$, Bertrand competition and Cournot competition yield the patentee the same profit.

It is easy to verify that both an auction and a license fee are optimal in the Bertrand model, as is royalty licensing. Indeed, the following can be easily established.

Proposition 10. Under Bertrand competition the three licensing policies are all equivalent for the patentee. His profit is $\varepsilon Q(c)$ if the innovation is not drastic and equals the monopoly profit under the new technology when the innovation is drastic. In the first case the equilibrium market price is c and in the latter case it is below c. Finally, Bertrand competition yields the patentee the same profit as under Cournot competition if $\varepsilon \geqq 2c/((n+1)\eta(c))$.

Appendix A

Proof of Lemma 1

(i) Differentiating (1a) with respect to k we have for $k \leqq K$

$$\frac{(\partial\eta/\partial p)(\partial p/\partial k)}{n\eta^2(p)} = \frac{-(\varepsilon/n)p-(\partial p/\partial k)(c-\varepsilon k/n)}{p^2}. \tag{A.1}$$

Suppose that $\partial p/\partial k \geqq 0$. Since $\partial\eta/\partial p > 0$, the left side of the equation is non-negative and the right side is negative. This contradiction implies that $\partial p/\partial k \leqq 0$. Suppose next that $k \geqq K$. Then, by (1b),

$$k(p-c+\varepsilon) = p/\eta.$$

Hence,

$$p-c+\varepsilon+k\frac{\partial p}{\partial k} = \frac{(\partial p/\partial k)\eta-(\partial\eta/\partial p)(\partial p/\partial k)p}{\eta^2}.$$

Therefore,

$$p-c+\varepsilon+\frac{\partial p}{\partial k}\left(k-\frac{1}{\eta}\right) = -\frac{\partial\eta}{\partial p}\frac{\partial p}{\partial k}\frac{p}{\eta^2}.$$

It is therefore sufficient to prove that $k \geqq 1/\eta$. Since $k \geqq K = c/\varepsilon\eta(c)$ it is sufficient that $c/(\varepsilon\eta(c) \geqq 1/\eta(p)$ or equivalently $\eta(p) \geqq (\varepsilon/c)\eta(c)$. The last inequality follows from the assumption that $\partial\eta/\partial p \geqq 0$ and from $\varepsilon \leqq c$. Now, as before, it follows that $\partial p/\partial k > 0$ cannot hold and that $\partial p/\partial k \leqq 0$.

(ii) The first-order condition of the firms' profit maximization are

$$p+P'q_i=c-\varepsilon, \quad i\in s,$$

$$p+P'q_j=c, \quad j\notin s,$$

provided that $k\leqq K$. Adding these n equations we have

$$np+P'Q=nc-k\varepsilon. \tag{A.2}$$

(iii) Differentiating both sides of (A.2) with respect to k we obtain

$$(n+1)\frac{\partial p}{\partial k}+\frac{\partial P'}{\partial k}Q=-\varepsilon,$$

Proof of Lemma 4

(i) From (6) we get that for each $k\geqq K$

$$\frac{\partial \Pi_0^a}{\partial k}(k)=\frac{\partial p}{\partial k}\left[Q(p)+\frac{p-c+\varepsilon}{P'}\right]=Q(p)\frac{\partial p}{\partial k}\left[1-\frac{\eta(p)}{p}(p-c+\varepsilon)\right].$$

Since the innovation is not drastic $K\geqq 1$, thus by (1b)

$$1-1/\eta(p)<1-1/(k\eta(p))=(c-\varepsilon)/p,$$

which implies that $(\eta(p)/p)(p-c+\varepsilon)<1$. Consequently, by Lemma 2 for $k>K$ $\partial\Pi_0^a(k)/\partial k<0$.

(ii) Suppose that $k\leqq K$ and $\varepsilon\geqq 2c/(n+1)\eta(c)$. Let

$$F(k)=k[\Pi^s(k)-\Pi^{n\setminus s}(k)].$$

Then by (3a) and (3b) for $k\leqq K$, $F(k)=-(\varepsilon k/P')(2p-2c+\varepsilon)$. Thus,

$$\frac{1}{\varepsilon}\frac{\partial F}{\partial k}=-\left[\frac{(2p-2c+\varepsilon+2k(\partial p/\partial k))P'-(\partial P'/\partial k)k(2p-2c+\varepsilon)}{(P')^2}\right].$$

Consequently,

$$\frac{\partial F}{\partial k}>0 \quad \text{iff} \quad \left(2p-2c+\varepsilon+2k\frac{\partial p}{\partial k}\right)P'-\frac{\partial P'}{\partial k}k(2p-2c+\varepsilon)<0.$$

By Lemma 1,

$$(n+1)\frac{\partial p}{\partial k}+\frac{\partial P'}{\partial k}Q=-\varepsilon. \tag{A.3}$$

This implies that

$$\frac{\partial F}{\partial k}>0 \quad \text{iff} \quad \left(2p-2c+\varepsilon+2k\frac{\partial p}{\partial k}\right)P'Q+\left[(n+1)\frac{\partial p}{\partial k}+\varepsilon\right]k(2p-2c+\varepsilon)<0.$$

By (A.2), $P'Q=-np+nc-k\varepsilon$. Hence,

$$\frac{\partial F}{\partial k}>0 \quad \text{iff} \quad \left(2p-2c+\varepsilon+2k\frac{\partial p}{\partial k}\right)(-np+nc-k\varepsilon)$$

$$+\left[(n+1)\frac{\partial p}{\partial k}+\varepsilon\right]k(2p-2c+\varepsilon)<0.$$

The last inequality is equivalent to

$$\frac{\partial p}{\partial k}[2k(p-c)-k\varepsilon(2k-n-1)]-n(p-c)[(2(p-c)+\varepsilon]<0. \tag{A.4}$$

Since $\partial p/\partial k<0$, this inequality holds whenever $n+1\geqq 2K$. Consequently, $\partial F/\partial k>0$ if $n+1\geqq 2c/(\varepsilon\eta(c))$ and the proof is complete.

Proof of Proposition 2

First let ε be such that $\varepsilon<c/(n\eta(c))$. This ensures that $n<K$ and $p>c$. Now, by (A.4), it is sufficient that

$$2(p-c)-\varepsilon(2k-n-1)>0. \tag{A.5}$$

The left side of (A.5) is a decreasing function of k. Hence, it is sufficient to prove (A.5) for the case where $k=n$. Namely,

$$2(\hat{p}-c)-(n-1)\varepsilon>0, \tag{A.6}$$

where $\hat{p}$ is the Cournot n-oligopoly price under the new technology $c-\varepsilon$. By the first-order condition,

$$1-1/(n\eta(\hat{p}))=(c-\varepsilon)/\hat{p},$$

it is easy to verify that $\hat{p}$ decreases with ε. Hence,

$$L(\varepsilon)=2(\hat{p}-c)-(n-1)\varepsilon$$

decreases with ε. Let

$$\bar{\varepsilon}=\sup\{\varepsilon \mid \varepsilon<c/(n\eta(c)), L(\varepsilon)>0\}.$$

Since $L(0)>0$, then $\bar{\varepsilon}>0$. Consequently, $\varepsilon\leqq\bar{\varepsilon}$ implies that $L(\varepsilon)>0$ and hence (A.6), (A.5) and (A.4) hold. This proves that the patentee's revenue increases for $k\leqq n-1$. Therefore, $k^*\geqq n-1$.

Proof of Proposition 5

(i) Since $w(k)=\Pi^s(k)$ for $k\geqq K+1$ we have by Lemma 2 that $w(k)$ is decreasing for $k\geqq K+1$. The optimal number of licensees is the largest k at which $w(k)$ decreases. Hence, $k\leqq K+1$ and the uniform fee is $\alpha=w(k)$.
(ii) The net profit of the firm (licensee or non-licensee) is $\Pi^{n\setminus s}(j)$ for some $0\leqq j\leqq n-1$. By Lemma 3, $\Pi^{n\setminus s}(j)\leqq\Pi^{n\setminus s}(0)$ where $\Pi^{n\setminus s}(0)$ is the pre-innovation profit of a firm. This level is achieved in equilibrium only if the innovation is licensed directly to one firm.
(iii) Let k^{f} be the optimal number of licenses under fee. Then

$$\Pi_0^{\mathrm{f}}=k^{\mathrm{f}}[\Pi^s(k^{\mathrm{f}})-\Pi^{n\setminus s}(k^{\mathrm{f}}-1)].$$

By part (i), $k^{\mathrm{f}}\leqq K+1$. If $k^{\mathrm{f}}\leqq K$ then by Lemma 2

$$\Pi_0^{\mathrm{f}}<k^{\mathrm{f}}[\Pi^s(k^{\mathrm{f}})-\Pi^{n\setminus s}(k^{\mathrm{f}})]<\Pi_0^{\mathrm{a}}.$$

If $k^{\mathrm{f}}=K+1$, then since $\Pi^{n\setminus s}(K)=0$, we have by Lemma 4 [part (i)]

$$\Pi_0^{\mathrm{f}}=(K+1)\Pi^s(K+1)<K\Pi^s(K)\leqq\Pi_0^{\mathrm{a}}.$$

Hence, $\Pi_0^{\mathrm{f}}<\Pi_0^{\mathrm{a}}$.

Proof of Proposition 6

To prove Proposition 6 we first establish:

Lemma 5. Consider a non-drastic innovation. Then the market price $p^*=p(r^*)$ under the optimal royalty satisfies $p^*>p(k)$ for any $1\leqq k\leqq K$, where $p(k)$ is the Cournot price of an industry with n firms and k licenses.

Proof. By (1a).

$$1-\frac{1}{n\eta}=\frac{c-\varepsilon+r}{p}. \tag{A.7}$$

Differentiating both sides of A.7 one obtains (after rearranging terms) that

$$\frac{\partial p}{\partial r}\left(1-\frac{1}{n\eta}+\frac{p(\partial\eta/\partial p)}{n\eta^2}\right)=1.$$

Hence by (A.7) and by Assumption 2 $\partial p/\partial r>0$ and thus $\partial Q/\partial r<0$. Hence, $q_\varepsilon \leqq q \leqq q_0$ where q_ε and q_0 are $Q(p(r))$ for $r=\varepsilon$ and $r=0$, respectively. Thus

$$\Pi_0^r=\max_{0\leqq r\leqq\varepsilon} rQ(p(r))=\max_{Q\varepsilon\leqq Q\leqq Q_0}\left[P(q)-c+\varepsilon+\frac{qP'}{n}\right]q. \tag{A.8}$$

Denote

$$g(q)=f(q)+q^2P'/n,$$

where

$$f(q)=(P(q)-c+\varepsilon)q.$$

Since

$$\partial g/\partial q=\partial f/\partial q+(q/n)(2P'+P''q)$$

we have by strict concavity of $qP(q)$ that

$$\partial g/\partial q<\partial f/\partial q \quad \text{for} \quad \text{any } q. \tag{A.9}$$

Let $q_m=Q(p_m)$ be the monopoly quantity under the new technology $c-\varepsilon$. Then $(\partial f/\partial q)(q_m)=0$. By the concavity of f, $(\partial f/\partial q)(q)<0$ for each $q>q_m$. Consequently by (A.9) $\partial g/\partial q<0$ for each $q>q_m$. This implies that the optimal solution q^* to (A.8) satisfies $q^*<q_m$. Consequently, $p^*>p_m$. Thus, it remains to prove that $p_m\geqq p(k)$ for each $1\leqq k\leqq K$. Observe that by (1a) and (1b)

$$p_m\left[\left(1-\frac{1}{\eta(p_m)}\right)\right]=c-\varepsilon, \tag{A.10}$$

and

$$p(k)\left[1-\frac{1}{n\eta(p(k))}\right]=c-\frac{k}{n}\varepsilon \quad \text{for } 0 \leqq k \leqq K. \tag{A.11}$$

Hence, by (A.10) and (A.11),

$$p_{\mathrm{m}}\left[1-\frac{1}{n\eta(p_{\mathrm{m}})}\right]=p(k)\left[1-\frac{1}{n\eta(p(k))}\right]+\frac{n-1}{n}\frac{p_{\mathrm{m}}}{\eta(p_{\mathrm{m}})}-\frac{n-k}{n}\varepsilon,$$

for each $0 \leqq k \leqq K$. Since $p[1-1/n\eta(p)]$ is an increasing function of p it remains to be proven that

$$\frac{n-1}{n}\frac{p_{\mathrm{m}}}{\eta(p_{\mathrm{m}})}-\frac{n-k}{n}\varepsilon \geqq 0,$$

for each $1 \leqq k \leqq K$. Thus, it is sufficient that $p_{\mathrm{m}}/\eta(p_{\mathrm{m}}) \geqq \varepsilon$ holds. The last inequality is equivalent to

$$-Q(p_{\mathrm{m}})/Q'(p_{\mathrm{m}}) \geqq \varepsilon. \tag{A.12}$$

Since $q_{\mathrm{m}}=Q(p_{\mathrm{m}})$ is the solution in p to $\partial f/\partial q=0$ we have

$$Q'(p_{\mathrm{m}})(p_m-c+\varepsilon)+Q(p_m)=0. \tag{A.13}$$

This is equivalent to

$$-Q(p_{\mathrm{m}})/Q'(p_{\mathrm{m}})=p_{\mathrm{m}}-c+\varepsilon.$$

Since the innovation is not drastic, $p_{\mathrm{m}} \geqq c$ and hence (A.12) holds. This completes the proof of Lemma 5 □.

Proof of Proposition 6. (i) The discussion following the definitions of the payoff functions establishes part (i).
(ii) The proof for this case of a non-drastic innovation follows directly from Lemma 5. Also, the proof of Lemma 5 establishes, in particular, that $p(r^*)>p_{\mathrm{m}}$. This together with Proposition 4, completes the proof of part (ii) for the drastic innovation case as well.
(iii) Suppose that the innovation is not drastic and that $n \geqq K$. Then by (3a) and (3b)

$$\Pi_0^{\mathrm{a}}=\max_{0 \leqq k \leqq K} k[\Pi^s(k)-\Pi^{n\setminus s}(k)]=\max_{0 \leqq k \leqq K} \frac{-\varepsilon k}{P'}(2p-2c+\varepsilon).$$

Consequently,

$$\Pi_0^{a} \geqq -\varepsilon K Q'(p(K))[2p(K)-2c+\varepsilon]. \tag{A.14}$$

On the other hand,

$$\Pi_0^{r} \max_{0 \leqq r \leqq \varepsilon} rQ(p(r)) = r^* Q(p(r^*))$$

where r^* is the optimal royalty for the patentee. Since $\varepsilon \geqq r^*$, it is sufficient to prove that

$$-KQ'(p(K))[2p(K)-2c+\varepsilon] > Q(p(r^*)). \tag{A.15}$$

By Lemma 5, $p(r^*) > p(K) = c$. Hence, $Q(c) > Q(p(r^*))$. Also, since $p(K) = c$ it is sufficient to prove that

$$-KQ'(c)\varepsilon \geqq Q(c). \tag{A.16}$$

Since $K = c/(\varepsilon\eta(c)) = Q(c)/(-\varepsilon Q'(c))$, (A.16) holds as an equality.

Suppose next that $n < K$. Then by Lemma 5 it is sufficient to prove, similar to (A.15), that

$$-nQ'(p(n))[2p(n)-2c+\varepsilon] > Q(p(n)).$$

It is easy to verify that the last inequality is equivalent to

$$1 - \frac{1}{n\eta(p(n))} \geqq -\frac{p(n)-c+\varepsilon}{p(n)}.$$

Using (1a) it is sufficient to prove that

$$\frac{c-\varepsilon}{p(n)} \geqq -\frac{p(n)-c+\varepsilon}{p(n)},$$

which is equivalent to $p(n) \geqq c$. Finally, it is easy to verify from (1a) that $\partial p/\partial n < 0$. Hence, $n \leqq K$ implies that $p(n) \geqq p(K) = c$ and the proof is complete for the non-drastic innovation case. Consider next a drastic innovation. By (A.8)

$$\Pi_0^{r} < \max_q \, [P(q) - c + \varepsilon]q = \Pi_{\mathrm{m}},$$

where Π_{m} is the monopoly profit under the new technology.

By Proposition 4, $\Pi_{\mathrm{m}} = \Pi_0^{a}$ and hence the proof of part (iii) is complete.

(iv) Suppose that for some c, say $\bar{c}$, $\eta(\bar{c})<1$. Under the royalty policy every firm's marginal cost is $\bar{c}-\varepsilon+r^*\leqq\bar{c}$. Hence it is sufficient to show that the total Cournot industry profit decreases with c for $\bar{c}-\varepsilon\leqq c\leqq\bar{c}$. Let

$$A=\frac{\partial}{\partial c}(Q(p-c))=\frac{\partial p}{\partial c}(Q'(p-c)+Q)-Q \tag{A.17}$$

for $\bar{c}-\varepsilon\leqq c\leqq\bar{c}$. For $\varepsilon=0$ and $k=n$ (1a) is equivalent to $n(p-c)+p'Q=0$. Hence $Q'=-Q/(n(p-c))$ and by (A.17)

$$A=\left[\frac{\partial p}{\partial c}\left(1-\frac{1}{n}\right)-1\right]Q.$$

Consequently $A<0$ if $\partial p/\partial c<n/(n-1)$. Eq. (1a) for $\varepsilon=0$ and $k=n$ is

$$\frac{c}{p}=1-\frac{1}{n\eta(p)}. \tag{A.18}$$

Differentiation of both sides of (A.18) w.r.t. c yields

$$\frac{\eta'(p)}{n\eta^2(p)}\frac{\partial p}{\partial c}=\frac{p-(\partial p/\partial c)c}{p^2}.$$

Since $\eta'(p)\geqq 0$ $\partial p/\partial c>0$. Hence $p-(\partial p/\partial c)c\geqq 0$ or $\partial p/\partial c\leqq p/c$. It is therefore left to show that $p/c<n/(n-1)$ for each $x, \bar{c}-\varepsilon\leqq c\leqq\bar{c}$. By (A.18) it is sufficient that $(n-1)/n>1-1/n\eta(p)$ or equivalently, $\eta(p)<1$. Since $\eta'(p)\geqq 0$ and $\eta(\bar{c})<1$ then $\eta(c)<1$, for $\bar{c}-\varepsilon\leqq c\leqq\bar{c}$. By (A.18) $p\to c$ as $n\to\infty$ uniformly in $[\bar{c}-\varepsilon,\bar{c}]$. Thus $\eta(p)<1$ for n sufficiently large. □

References

Arrow, K., 1962, Economic welfare and the allocation of resources for invention, in: R.R. Nelson, ed., The rate and direction of inventive activity (Princeton University Press, Princeton, NJ).

Kamien, M.I., 1990, Patent licensing, in: R.J. Aumann and S. Hart, eds., Handbook of game theory with economic applications, forthcoming.

Kamien, M.I. and Y. Tauman, 1984, The private value of a patent: A game theoretic analysis, Journal of Economics 4, 93–118.

Kamien, M.I. and Y. Tauman, 1986, Fee versus royalties and the private value of a patent, Quarterly Journal of Economics 101, 471–491.

Kamien, M.I., Y. Tauman and S. Zamir, 1990, The value of information in a strategic conflict, Games and Economic Behavior 2, 129–153.

Kamien, M.I., Y. Tauman and I. Zang, 1988, Optimal license fees for a new product, Mathematical Social Sciences 16, 77–106.

Katz, M.L. and C. Shapiro, 1985, On the licensing of innovation, Rand Journal of Economics 16, 504–520.

Katz, M.L. and C. Shapiro, 1986, How to license intangible property, Quarterly Journal of Economics 101, 567–590.

Stobiecki, M., 1975, Obtaining the desired partition into priority classes in queues by prices, M.Sc. thesis (Faculty of Management, Tel Aviv University, Tel Aviv).

Tauman, Y. and Y. Weiss, 1990, Licensing of innovations under incomplete information, Working paper no. 90-19 (Institute for Decision Sciences, State University of New York).