A Comparative Study of Stochastic Unit Commitment and Security-Constrained Unit Commitment Using High Performance Computing

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Abstract— **The large-scale integration of renewable resources has recently raised interest in systematic methods for committing locational reserves in order to secure the system against contingencies and the unpredictable and highly variable fluctuation of renewable energy supply, while accounting for power flow constraints imposed by the transmission network. In this paper we compare two approaches for committing locational reserves: stochastic unit commitment and a hybrid approach of scenario-based security-constrained commitment. Parallel algorithms are developed for solving the resulting models, based on Lagrangian relaxation and Benders decomposition. The proposed algorithms are implemented in a high performance computing environment and the performance of the resulting policies is tested against a reduced model of the California ISO interconnected with the Western Electricity Coordinating Council.**

I. Introduction

The increasing uncertainty of power system operations due to the large-scale integration of renewable energy resources and demand response has raised an interest in systematic methods for committing day-ahead reserves in order to operate the system reliably within an uncertain environment. The challenge of committing reserves lies in optimizing the tradeoff between system security and economic operation of the system. Any level of security can be achieved in the system given a sufficient amount of reserves. The challenge rests in choosing the level of reserves that satisfies certain operational criteria in an uncertain environment at least cost. Traditional reserve commitment approaches [1] rely on reserve requirements and security constraints that are meant to mitigate continuous fluctuations in demand and renewable supply as well as discrete disturbances such as generator and transmission line failures. However, these models often fail to capture the full range of complexity in an uncertain environment and rely instead on heuristic practices adopted by operators through experience.

The power system operations literature has proposed four fundamental paradigms for representing uncertainty and optimizing the commitment of reserves at least cost: stochastic optimization, security-constrained approaches, robust optimization and probabilistic constraints. Stochastic programming was originally posed in the context of unit commitment by Takriti and Birge [2] as an approach for mitigating demand uncertainty and generator outages. Subsequently, numerous variants of the stochastic unit commitment model have been proposed [3], [4], [5], [6], [7], [8], [9] that vary based on the number of stages, the source of uncertainty, the representation of uncertainty and solution methods that are used. The drawback of stochastic unit commitment is the requirement to represent uncertainty in a detailed fashion, by using a large number of appropriately weighted scenarios. Generating these scenarios requires detailed information about the stochastic process that generates uncertainty in the system (e.g. time series models or failure rates). Moreover, the choice of how many scenarios to select, which ones to select and how to weigh them is non-obvious and can have a significant impact on the performance of the resulting commitment. Moreover, the resulting problems are large-scale and Lagrangian relaxation is often employed in order to decompose the problem to tractable subproblems.

Security-constrained unit commitment models require that the system be capable of withstanding major element failures without shedding load. Security constraints address discrete failures of network elements, whereas continuous sources of uncertainty are addressed either through scenarios or exogenous reserve criteria. Wang et al. [10] account for supply and demand fluctuations through exogenous reserve criteria [10] and use Bender's decomposition to solve the problem, while Wu et al. [11] account for continuous sources of disturbances through scenarios and use Lagrangian relaxation in order to solve the problem.

Two additional systematic approaches to short-term scheduling under uncertainty that are not explored in this paper include robust optimization and probabilistic constraints. In robust optimization models [12], [13] the objective is to commit reserves in order to minimize the cost of operating the system against the worst-case realization of uncertainty. The worst-case nature of these approaches reflects more closely the tendency of operators to operate the system in a conservative fashion. These models exploit Benders' decomposition in order to solve the resulting bilevel programming problems. Probabilistically constrained models [14], [15] commit units in order to ensure the satisfaction of demand within an exogenously defined probability. Both robust optimization and probabilistically constrained models share the advantage that they require only limited information about the process that drives uncertainty in operations.

The methods discussed above increase the computational challenges associated with unit commitment problems. The solution methods used for solving short-term scheduling problems can often be parallelized. As a result, these applications invite the use of distributed computing for addressing the challenge of short-term scheduling under uncertainty, in much the same way that mid-term scheduling under uncertainty has thrived in the power system operations community over the past decades.

Distributed computation has a rich history in the area

of power systems planning and operations. A review of the application of high performance computing in power systems is presented by Falcao [16]. Falcao presents various applications of parallelization, including security-constrained optimal power flow and composite generation-transmission reliability evaluation. Pereira et al. [17] present the application of distributed computing in reliability evaluation for composite outages, scenario analysis for hydro dominated systems and security-constrained dispatch. Monticelli et al. [18] formulate the security-constrained optimal power flow with corrective rescheduling and demonstrate economic benefits in the dispatch on an IEEE system with 118 buses. Although the authors observe the possibility of parallelizing their algorithm, they do not present such a parallel implementation. Kim and Baldick [19] present a parallel algorithm for solving distributed optimal power flow. The authors present efficiency and speedup results, although these are estimated as the authors do not implement the algorithm in parallel. Bakirtzis and Biskas [20] propose a decentralized Lagrangian relaxation algorithm for solving the optimal power flow problem presented by Kim and Baldick [19]. The authors test three test systems, including a 3-area version of the IEEE RTS 96 system, a scaled-up version of the latter with more areas and a full model of the Balkan system. A parallel implementation of the algorithm in Bakirtzis and Biskas [20] using PVM is presented by Biskas et al. [21].

Despite the fact that there is a rich body of literature focused on short-term scheduling under uncertainty, the relative performance of the models proposed in the literature is not compared adequately in order to appreciate the tradeoffs involved in using each paradigm. This paper serves two purposes. The first objective is to motivate this discussion by presenting a comparative study of stochastic programming and security-constrained unit commitment models. The second objective is to demonstrate the benefits of distributed computation in accelerating the solution of these models, and validate the great promise that high performance computing and cloud computing hold for the short-term scheduling of smart grids under conditions of large-scale renewable energy and demand response integration. We present our models in Section II and decomposition methods for solving these models in Section III. A case study of the California ISO interconnected with the Western Electricity Coordinating Council is presented in Section IV. We summarize our conclusions in Section V.

II. Model Description

The problem that this paper is focusing on is the day-ahead scheduling of generators subject to real-time renewable power supply uncertainty and outages of transmission lines and generators. The problem is cast as a two-stage optimization, where the first stage represents day-ahead decisions and the second stage represents the real-time recourse to the revealed system conditions.

In the following model formulation, u represents a binary variable indicating the on-off status of a generator, v is a binary startup variable and p is the production level of each generator. The minimum load cost of a generator is denoted as K_g, the startup cost as S_g and the constant marginal cost as C_g. The model that we present in this paper accounts for transmission constraints, with power flows over transmission lines denoted as e. The demand for each hour t at each bus of the network n is denoted as D_{nt}. Operating constrains are denoted compactly in terms of a feasible set $\mathcal{D}$, and vectors are denoted in bold. Thus, the notation $(\mathbf{p}, \mathbf{e}, \mathbf{u}, \mathbf{v}) \in \mathcal{D}$ encapsulates the minimum/maximum run limits, minimum up/down times and ramping rate limits of generators, as well as Kirchhoff's voltage and current laws and the thermal limits of lines.

The objective is to minimize the cost of serving forecast demand. The problem in the deterministic setting (assuming an accurate forecast of renewable power production and demand) can be described as follows:

$$
\begin{aligned}
&(UC): \min \sum_{g\in G}\sum_{t\in T}(K_g u_{gt} + S_g v_{gt} + C_g p_{gt}) \\
&s.t. \sum_{g\in G_n} p_{gt} = D_{nt} \\
&P_g^- u_{gt} \le p_{gt} \le P_g^+ u_{gt} \\
&e_{lt} = B_l(\theta_{nt} - \theta_{mt}) \\
&(\mathbf{p}, \mathbf{e}, \mathbf{u}, \mathbf{v}) \in \mathcal{D}, \qquad (1)
\end{aligned}
$$

The set of generators located in each bus n is denoted by G_n. The horizon T is 24 hours, with hourly increments. A detailed formulation of the constraints represented by the domain $\mathcal{D}$ can be found in Papavasiliou and Oren [22].

A. Stochastic Unit Commitment

The stochastic formulation follows the model of Ruiz et al. [6] and involves a two-stage process, where the set of uncertain outcomes is represented as S. First-stage unit commitment and startup decisions are represented respectively as w and z and apply for those generators G_s for which commitment decisions need to be made in advance, in the day-ahead time frame. The problem to be solved is the following:

$$
\begin{aligned}
&(SUC): \\
&\min \sum_{g\in G}\sum_{s\in S}\sum_{t\in T} \pi_s(K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\
&s.t. \sum_{g\in G_n} p_{gst} = D_{nst}, \\
&P_{gs}^- u_{gst} \le p_{gst} \le P_{gs}^+ u_{gst} \\
&e_{lst} = B_{ls}(\theta_{nst} - \theta_{mst}) \\
&(\mathbf{p}, \mathbf{e}, \mathbf{u}, \mathbf{v}) \in \mathcal{D}_s \\
&u_{gst} = w_{gt}, v_{gst} = z_{gt}, \qquad (2)
\end{aligned}
$$

where decision variables are now contingent on the scenario $s \in S$. Note that the domain $\mathcal{D} = \times_{s\in S}\mathcal{D}_s$ is decomposable across scenarios. Scenarios represent the realization of hourly renewable supply production, which results in uncertain net

demand D_{nst} in each bus, as well as the loss of generators for the entire day (in which case the capacity limits of a generator are $P^-_{gs} = P^+_{gs} = 0$, which forces a unit to produce zero output), and the loss of lines (in which case the susceptance of a line is $B_{ls} = 0$, which forces power flow over the line to equal zero).

B. Scenario-Based Security-Constrained Unit Commitment

Note that load shedding is permitted in the stochastic unit commitment model of Eq. (2), with lost load incurring a high penalty in the objective function. Loads L are therefore represented as a dummy generator with second-stage production decisions p_{lst} and a marginal cost equal to the value of lost load. As a result, the feasible region of each scenario, $\mathcal{D}_s$, is non-empty for any choice of first-stage decision variables w_{gt}, z_{gt}.

In a security-constrained model discrete disturbances are accounted for by requiring that the system be capable of withstanding any element failure. This implies that each scenario s now consists of at most a single contingency. Following the model of Wu et al. [11], we account for continuous disturbances (net demand forecast errors) by associating a renewable supply outcome with each scenario s rather than imposing exogenous reserve requirements. Scenarios that involve no contingency are weighed with a positive probability in the objective function π_s of Eq. (3), whereas scenarios that involve contingencies are only included in the constraint set. The feasible region is equal to $\mathcal{D}_s$ with the additional constraint that $p_{lst} = 0$ for load shedding, and in contrast to (SUC) there may be choices of first-stage decisions for which the model is infeasible (i.e. $\{\mathcal{D}_s, p_{lst} = 0, u_{gst} = w_{gt}, v_{gst} = z_{gt}\} = \emptyset$).

$$
\begin{aligned}
&(SCUC): \\
&\min \sum_{g\in G}\sum_{s\in S}\sum_{t\in T} \pi_s(K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\
&s.t. \sum_{g\in G_n} p_{gst} = D_{nst}, \\
&P^-_{gs} u_{gst} \le p_{gst} \le P^+_{gs} u_{gst} \\
&e_{lst} = B_{ls}(\theta_{nst} - \theta_{mst}) \\
&(\mathbf{p}, \mathbf{e}, \mathbf{u}, \mathbf{v}) \in \mathcal{D}_s \\
&p_{lst} = 0, l \in L \\
&u_{gst} = w_{gt}, v_{gst} = z_{gt}, \qquad (3)
\end{aligned}
$$

C. Scenario Selection

The selection of scenarios in the stochastic unit commitment model of Section II-A is based on an idea inspired by importance sampling [22]. A large number of candidate scenarios $\omega \in \Omega$ are evaluated in terms of their cost impact to the system $C_D(\omega)$, where this cost impact is evaluated against an easily computable deterministic unit commitment model. Candidate scenarios are then selected to enter the set of selected scenarios S by sampling according to a probability which is proportional to their cost impact. These scenarios are assigned a weight π_s in the objective function of (SUC) which is inversely proportional to their cost impact $C_D(\omega)$ in order to un-bias their selection. Note that in the stochastic programming formulation, a scenario s can involve any number of contingencies and not necessarily a single contingency.

In the case of the scenario-based security-constrained model, the set S is generated by the Cartesian product of a set of renewable supply outcomes with the no-contingency outcome and the most severe single-element contingencies in the system. The set of scenarios that involve the no-contingency outcome are assigned an equal positive probability in the objective function, whereas the scenarios involving single-element contingencies have no direct impact on the objective function through their weight, $\pi_s = 0$, but only through their presence in the constraint set.

III. Solution Methodology

In the following section we present two decomposition methods for solving (SUC) and $(SCUC)$, as well as distributed implementations of the decomposition algorithms.

A. Lagrangian Relaxation

The Lagrangian relaxation algorithm relies on the observation that the relaxation of the non-anticipativity constraints in (SUC) results in unit commitment subproblems that are independent across scenarios. The Lagrangian dual function is obtained as:

$$
\begin{aligned}
\mathcal{L} = &\sum_{s\in S} \pi_s \Big(\sum_{g\in G}\sum_{t\in T} (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\
&+ \sum_{g\in G_s}\sum_{t\in T} (\mu_{gst}(u_{gst} - w_{gt}) + \nu_{gst}(v_{gst} - z_{gt})) \Big) \qquad (4)
\end{aligned}
$$

The problem is solved by maximizing the Lagrangian dual function using the sub-gradient algorithm. The solution of the Lagrangian involves one second-stage unit commitment problem for each scenario $(P2_s)$, and one first-stage optimization $(P1)$. The first-stage optimization is formulated as:

$$
\begin{aligned}
&(P1): \max \sum_{g\in G_s}\sum_{s\in S}\sum_{t\in T} \pi_s(\mu_{gst} w_{gt} + \nu_{gst} z_{gt}) \\
&s.t. \mathcal{D}_1, \qquad (5)
\end{aligned}
$$

where $\mathcal{D}_1$ represents the minimum up and down time constraints of slow units $g \in G_s$.

The solution of the Lagrangian dual provides a lower bound for the model. By introducing redundant second-stage decision variables on startup decisions, we are able to enforce minimum up and down times on slow units, as in Eq. (5). Given these unit commitment schedules, we can solve an economic dispatch model (ED_s), which is $(P2_s)$ with u_{gst}, v_{gst} fixed for $g \in G_s$. This provides an upper bound that can be used for obtaining feasible solutions at each iteration as well as a duality gap. This duality gap is used as a termination criterion. The algorithm is parallelized both in the solution of $(P2_s)$, as well as the solution of (ED_s), as indicated in Fig. 1. Further details about the

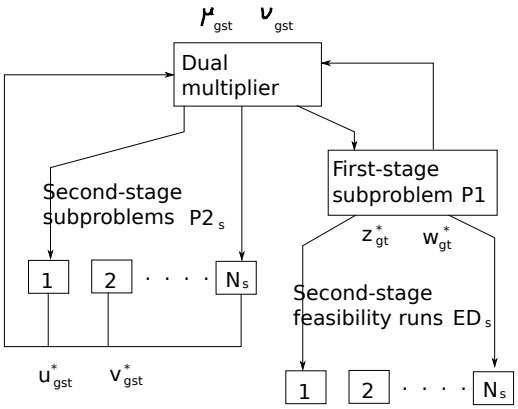

Fig. 1. The parallel implementation of the Lagrangian relaxation algorithm.

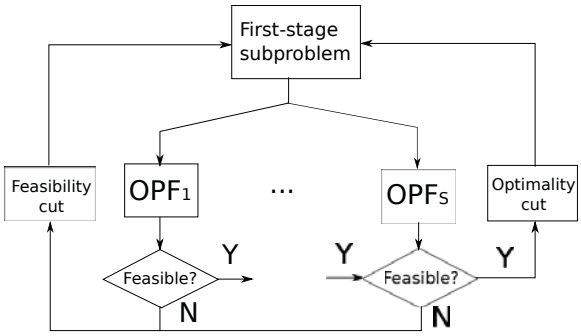

Fig. 2. The parallel implementation of the Benders decomposition algorithm.

solution methodology are discussed in Papavasiliou et al. [5].

B. Benders Decomposition

Security constraints are enforced in power system operations in order to protect the system against the failure of any given transmission or generation element. The security constraints require that the system be capable of withstanding the loss of any single component in the system while fully satisfying demand. Security-constrained unit commitment can be approximated as a special case of the (SUC) model presented in Eq. (2) when $\pi_s > 0$ for the no-contingency scenarios, and $\pi_s = 0$ for scenarios involving contingencies. This implies that the constraints associated with each contingency scenario are enforced in the constraint set, but are not weighed in the objective function. This remains an approximation of $(SCUC)$ since the constraint p_{lst}, $l \in L$, is not enforced in (SUC).

In principle, this approximation of the security-constrained unit commitment problem can be solved by using the solution algorithm of Section III-A. In practice this approach presents convergence problems when solved by Lagrangian relaxation. The dual function is not increasing even when the step size is reduced to a very small amount, and the unit commitment schedule of slow generators is inverted after each iteration. This numerical instability is due to the fact that the dual function is very steep, which results from the fact that the operating cost terms vanish in Eq. 4 since $\pi_s = 0$ for the scenarios associated with contingencies.

This motivates a Benders decomposition scheme for solving the problem. This can be justified by the fact that all feasibility constraints can be satisfied with only a few feasibility cuts associated with the most severe contingencies in the system. Optimality cuts can be defined by solving only those few scenarios associated with the no-contingency outcome. The advantage of using a Benders decomposition scheme is that the generation of feasibility cuts and optimality cuts can be parallelized, which implies that the second stage of the model is no more the computational bottleneck.

The algorithm that we propose in this paper requires two assumptions:

Assumption 1:. In order to maintain the convexity of the second-stage value function, it is necessary to assume that second-stage problems are continuous. Therefore, we impose the assumption that unit commitment decisions have to be fixed for all generators in the network from the first stage. This is contrasted to the Lagrangian relaxation algorithm that can involve integer decisions in the second stage for fast generators $g \in G_f = G - G_s$.

Assumption 2: The generation of feasibility cuts according to Van-Slyke and Wets [23] removes one candidate integer solution at each iteration, however this process can easily stall when there is a large number of candidate integer solution combinations that need to be tested before a feasible solution can be obtained, as is the case in the stochastic unit commitment problem. By assuming away ramping constraints in $(SCUC)$, we obtain a feasible region $(\mathcal{D}_{st})$ that is decomposable both by time period as well as scenario. Rather than using the feasibility cuts of Van-Slyke and Wets [23], we then impose the constraints represented by $\mathcal{D}_{st}$ in the first-stage problem for the scenario and time period that represents the most severe contingency given the current candidate integer solution. The motivation is that accounting for the most severe contingency in the first stage of the problem is capable of satisfying most operating constraints associated with less severe contingencies. The algorithm is presented in Fig. 2.

IV. Results

In this section we analyze a test system of the California Independent System Operator interconnected with the Western Electricity Coordinating Council. The system is composed of 225 buses, 375 lines and 130 generators. The fuel mix of the generators and their classification among fast and slow units is shown in Table I. The schematic of the system under consideration is presented in Fig. 3. The value of lost load is assumed equal to 5,000$/MWh [5].

TABLE I

GENERATION MIX FOR THE TEST CASE

Type	No. of units	Capacity (MW)
Nuclear	2	4,499
Gas	88	18,745.6
Coal	6	285.9
Oil	5	252
Dual fuel	23	4,599
Import	22	12,691
Hydro	6	10,842
Biomass	3	558
Geothermal	2	1,193
Wind (moderate)	5	6,688
Wind (deep)	10	14,143
Fast thermal	82	9,156.1
Slow thermal	42	19,225.4

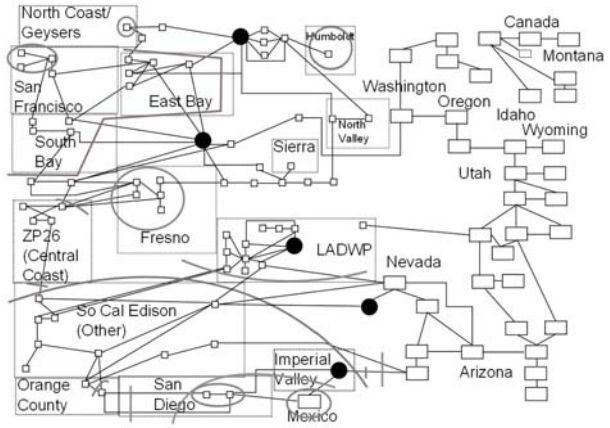

Fig. 3. A schematic of the WECC model studied in the Results section.

The wind penetration level that we analyze corresponds to the 2030 wind integration targets of California for a typical spring weekday. The wind model is calibrated against one year of data from the National Renewable Energy Laboratory. The wind power production time series model is described in detail by Papavasiliou and Oren [24]. The model captures temporal correlations of wind speed, the nonlinear conversion of wind speed to wind power, the locational correlations of the wind sites under consideration, as well as systematic seasonal and diurnal characteristics of the data set. The wind power production time series model was used both in order to generate scenarios for the unit commitment optimization models, as well as for generating outcomes for the Monte Carlo simulation of the performance of the two different unit commitment policies. The set of outcomes that were used for the Monte Carlo performance evaluation were different from the scenarios that were used as input to the unit commitment models.

Both formulations were solved for 30 scenarios in order to compare the two models on a fair basis. The scenario selection algorithm of Section II-C was used for selecting and weighing the scenarios of (SUC). The input for the $(SCUC)$ model was generated from the Cartesian product of ten wind power production scenarios with the no-contingency

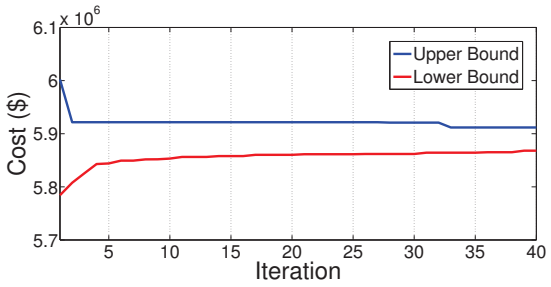

Fig. 4. The evolution of the Lagrangian Relaxation algorithm bounds.

case as well as the two most severe contingencies in the network, namely the failure of each of the two nuclear units in the network. Both algorithms were implemented in the Java callable library of CPLEX 12.4, and parallelized using the Message Passing Interface (MPI). The code was implemented on a high performance computing cluster in the Lawrence Livermore National Laboratory on a network of 1,152 nodes, 2.4 GHz, with 8 CPUs per node and 10 GB per node.

Each unit commitment policy was evaluated against 1,000 Monte Carlo outcomes of wind power production and contingencies. We assume a probability of generator failure equal to 1% [25] and a probability of transmission line failure equal to 0.1% [26].

For the implementation of the (SUC) algorithm described in Fig. 1, $(P1)$ and $(P2_s)$ were run for 80 iterations. For the last 40 iterations, (ED_s) was run for each scenario in order to obtain a feasible solution and an upper bound to the problem. The evolution of the duality gap is shown in Fig. 4.

The Benders decomposition algorithm required 31 iterations to converge. During these iterations, either feasibility cuts were added to the first-stage program, or a new approximation of the value function was generated, along with an estimate of the gap in the current candidate unit commitment solution. The evolution of the gap is shown in Fig. 5, with zeros representing an iteration at which a feasibility cut was added. Note that the first feasible unit commitment schedule is detected in iteration 19. The gap at iteration 19, which cannot be shown in the bounds of Fig. 5, is equal to 5.855 $M. Subsequently, the value function approximation improves around the neighborhood of the optimal solution, and although 4 more feasibility cuts are added in the remaining iterations, the algorithm eventually terminates after 31 iterations.

A. Relative Peformance

The hourly day-ahead capacity committed by each model in each hour of the day is shown in Fig. 6. We note that the $(SCUC)$ model is committing significantly more capacity than the (SUC) model. This can be attributed to Assumption 2 of Section III-B. Due to the fact that the Benders decomposition algorithm requires that all units be committed in the day-ahead time frame, the resulting

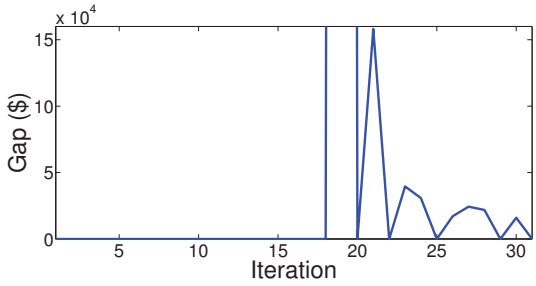

Fig. 5. The evolution of the Benders Decomposition gap.

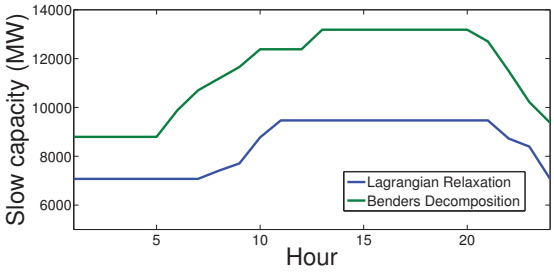

Fig. 6. Hourly day-ahead capacity committed by each model.

TABLE II
DAILY OPERATING COST ($1,000)

	Startup	Min. load	Load shed	Fuel	Total
Benders	66.5	1,205.3	0	4687.3	5,959.1
LR	106.0	699.4	0.3	4,831.5	5,637.2

policy is quite conservative. The cost performance of the two approaches in the Mote Carlo simulation is shown in Table II. We note that the (SUC) model outperforms the $(SCUC)$ model by 5.4% relative to the average daily cost of the $(SCUC)$ model in terms of expected cost performance.

It is interesting to note that both models are outperforming each other relative to the objectives that they are optimizing. The (SUC) model is outperforming $(SCUC)$ in terms of expected cost performance, while the $(SCUC)$ model achieves zero load shedding, as we demonstrate in Table II, where we note that the (SUC) model is shedding small quantities of load. The tradeoff for the increased reliability of the $(SCUC)$ model is the over-commitment of day-ahead capacity, which reduces the operational flexibility of the system in real time, resulting in excessive startup, minimum load and fuel costs.

B. Running Time

The running time of the Benders decomposition algorithm is shown in Fig. 7. The speedup of the algorithm is due to the parallelization of the continuous DC optimal power flow problems that are required for generating feasibility and optimality cuts (see Fig. 2). The marginal benefits vanish beyond 15 processors. The entire model requires 26.6 minutes to solve in a fully serial implementation, versus 14.8 minutes in a fully parallel implementation. We note that the benefits of parallelism are expected to increase as we increase the number of contingencies or wind scenarios considered in the model. However, as an excessive number of second-stage problems is added to the $(SCUC)$ model, additional feasibility cuts are required in order to generate feasible unit commitment schedules. This may result in a non-decomposable first-stage problem that is excessively large, and for which distributed computation can offer no speedup benefits. In that case, the first-stage problem will dominate the total running time of the problem. The motivation of using Benders decomposition in unit commitment problems is that the most severe contingencies in the system often suffice for withstanding most minor contingencies. However, Van Slyke and Wets [23] note the possibility that the Benders decomposition algorithm may not suffice to solve the problem if an excessive number of feasibility cuts are required. We have encountered this behavior in an instance of the $(SCUC)$ problem with 100 contingencies (which results in 1,000 scenarios when the contingencies are interleaved with 10 wind scenarios), and in future research we intend to explore alternative approaches for solving larger instances of the problem.

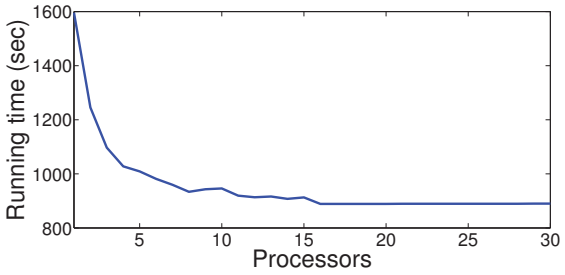

Fig. 7. The running time of the Benders decomposition algorithm as a function of processor number.

The running time of the Lagrangian relaxation algorithm is shown in Fig. 8. The marginal benefits of parallelization vanish beyond 15 processors. The solution time of the Lagrangian relaxation algorithm ranges between 15.8 hours for the fully serial implementation to 47.7 minutes in the fully parallel implementation. The benefits of parallelization are evident in this example, as they enable us to reduce the solving time of the original problem to a time horizon that is acceptable for operational purposes. In contrast to the $(SCUC)$ model, the proposed Lagrangian relaxation can scale to a very large number of scenarios provided that a sufficient number of processors is available.

V. CONCLUSIONS

We present two approaches for solving the unit commitment problem in order to mitigate the uncertainty stemming from continuous sources of uncertainty (renewable energy or demand forecast error) as well as discrete disturbances (generator and transmission line failures). The stochastic unit commitment model optimizes the expected cost of

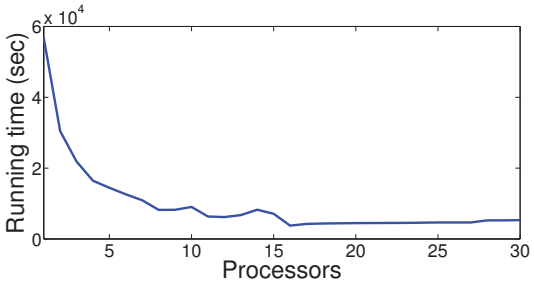

Fig. 8. The running time of the Lagrangian relaxation algorithm as a function of processor number.

operation of the system, while the scenario-based security constrained unit commitment model minimizes the cost of system operations while guaranteeing that the system can withstand major contingencies without shedding load. We present a Lagrangian relaxation algorithm for solving the stochastic unit commitment model and a Benders decomposition algorithm for solving the security-constrained unit commitment model and we implement both algorithms in a high performance computing environment. The Benders decomposition algorithm is implemented by passing power flow constraints associated to the most severe contingencies in the system to the first-stage problem.

We compare the two approaches on a test case of the California ISO interconnected with the Western Electricity Coordinating Council. We observe that the security-constrained model commits significantly greater quantities of day-ahead capacity and outperforms the stochastic unit commitment model in terms of load shedding. Instead, the stochastic unit commitment model outperforms the security-constrained model in terms of expected cost by reducing minimum load, startup and fuel costs. We also find that the parallel implementation of the stochastic unit commitment problem reduces the running time of the model to a level that is acceptable for operational purposes. The Benders algorithm also benefits from parallelization, although running time in the Benders algorithm is dictated by the first-stage subproblem. In contrast to the Lagrangian relaxation algorithm which can scale to a very large number of scenarios provided a sufficient number of processors is available, further research is required in order to solve larger instances of the security-constrained model.

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