

Managing Risk under a Fixed Price Load Following Obligation for Electricity Service

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Abstract— Load serving entities (LSE) and holders of default service obligations, in restructured electricity markets, provide electricity service at regulated or contracted fixed prices while facing wholesale procurement cost or opportunity cost at volatile spot prices. We address the hedging problem of such entities which face joint price and quantity risk. Exploiting the correlation between consumption quantities and spot prices, we develop optimal, self-financed hedging strategies under various assumptions regarding the uncertainty in wholesale price and consumption volume and under different models of risk aversion and hedging objectives. We then show how such strategies can be implemented using standard forward contracts and commodity derivatives.

Keywords: Electricity Markets, Risk Management, Volumetric hedging,

I. INTRODUCTION

The introduction of competitive wholesale markets in the electricity industry has put high price risk on market participants, particularly on load serving entities (LSEs). The unique non-storable nature of electricity as a commodity eliminates the buffering effect associated with holding inventory and makes the possibility of sudden large price changes more likely. Significant market risks that are faced by LSEs are not related to price alone. Volumetric risk (or quantity risk), caused by uncertainty in the electricity load is also an important exposure for LSEs since they are obligated to serve the varying demand of their customers at fixed regulated prices. Electricity consumption volume directly affects the company's net earnings and more importantly the spot price itself. A similar situation is faced by holders of default service contracts (e.g. in New Jersey [1]) who undertake through an auction, an obligation to serve a fixed percentage of retail load over a predetermined period at a fixed rate. Standard hedging strategies that only concern price risks for a fixed amount of volume cannot fully hedge market risks faced by such entities who have a load following obligation. The price and volumetric risks are especially severe because supply and demand conditions usually shift adversely together as demonstrated by the California electricity crisis in 2000 and 2001, which led three large

investor owned utilities in California to bankruptcy or near bankruptcy.

As a way of mitigating price risks in electricity markets, derivatives such as futures, forwards and options have been used. An electricity forward contract obligates a party to buy and the counter party to sell a specified quantity on a given future date at a predetermined fixed price. At the maturity date if the market price is higher than the contracted forward price, then the buyer would profit, conversely, if the market price is lower than the forward price then the buyer will suffer a loss. The profits and losses are settled when the delivery is completed. Put or call options are also used for different types of risk hedging: A “European style” call (put) option on electricity supply entitles the holder the right to buy (sell) a specified quantity of electricity at a specified delivery time at an agreed upon “strike” price. However, when electricity deliveries are handled through the ISO wholesale market, such contracts are settled financially as contracts for differences (CFD) which entitle the option holders to receive from the option sellers the difference between the wholesale spot price and the contract strike price. A call (put) option is said to be “in the money” (as opposed to out of the money) and entitled to compensation when the spot price is above (below) the strike price. LSEs can use such call options to avoid the risk of higher prices while still being able to forgo the contract and enjoy the benefit of lower spot prices when they occur.

While it is relatively simple to hedge price risks for a specific quantity, such hedging becomes difficult when the demand quantity is uncertain, i.e., volumetric risks are involved. In the presence of volumetric risk a company should be hedged against fluctuations in total profit (i.e., price minus cost times quantity) but unfortunately, there are no simple market instruments that would enable such hedging. Furthermore, the common approach of dealing with demand fluctuations for commodities by means of inventories is not possible in electricity markets where the underlying commodity is not storable.

The non-storability of electricity combined with the steeply rising supply function and long lead time for capacity expansion results in strong positive correlation between demand and price. When demand is high, due to a heat wave, the spot prices will be high as well and vice versa. For example, the correlation coefficient between hourly price and load for two years from April 1998 in California² was 0.539.

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² During this period, all the regulated utilities in the California market procured electricity from the spot market at the Power Exchange (PX). They were deterred from entering into long term contracts through direct limitations on contract prices and disincentives due to ex post prudence requirements.

[2] also calculated the correlation coefficients between normalized average weekday price and load for 13 markets: for example, 0.70 for Spain, 0.58 for Britain, and 0.53 for Scandinavia. There are some markets where this price and load relationship is weak but in most markets load is the most important factor affecting price of electricity.

The correlation between load and price amplifies the exposure of an LSE having to serve the varying demand at fixed regulated prices and accentuates the need for volumetric risk hedging. An LSE purchasing a forward contract for a fixed quantity at a fixed price based on the forecasted demand quantity will find that when demand exceeds its forecast and contracted position is short then the spot price will be high and most likely will exceed its regulated sale price, resulting in losses. Likewise, when demand is low below its forecast, the spot price at which the LSE will have to sell its surplus will be low and most likely below its purchase price, again resulting in losses.

Because of the strong causal relationship between electricity consumption and temperature, weather derivatives have been considered to be an effective means of hedging volumetric risks in the electricity market. Weather derivatives, whose payoff is triggered by weather conditions, exploit the correlation between electricity demand and weather condition. For example, if the upcoming winter is milder than usual, the electricity demand would be low leaving an LSE with low revenue. The LSE can protect against such situation by buying a Heating Degree Days (HDD) put option, which has a positive payoff if the weather turns out to be milder than the strike value of the HDD (and zero otherwise). However, the speculative image of such instruments makes them undesirable for a regulated utility having to justify its risk management practices and the cost associated with such practices to a regulator.

We propose an alternative to weather derivatives which involves the use of standard forward electricity contracts and price based power derivatives. This approach to volumetric hedging exploits the correlation between load and price. Electricity markets are generally incomplete markets in the sense that not every risk factor can be perfectly hedged by market traded instruments. In particular, the volumetric risks are not traded in the electricity markets. Thus, we cannot naively adopt the classical no-arbitrage approach for hedging volumetric risks. Our proposed methodology is based on the alternative approach offered by the economic literature for dealing with risks that are not priced in the market, by considering the utility of economic agents bearing such risks.

This presentation contains an overview of the problem described above and summarize the results of work reported in [3], [4] and [5] which also contain an extensive review of related literature. The first two papers focus on a single period setting where the hedging portfolio is assembled at the beginning of the hedging period and liquidated at the end of the period which occurs at the time when demand is realized and energy is delivered. In [5] we also explore the optimal timing of the portfolio assembly within the hedging period, extending earlier work reported in [6].

There are two basic approaches to creating an optimal hedging portfolio. The first approach, proposed in [7], is to determining the optimal number of contracts from a set of

available contract choices. This approach requires the solution of a difficult numerical optimization problem and provides little insight regarding the overall hedging strategy. We use an alternative approach by first determining a continuous optimal payoff function that represents payoff of a hedging portfolio as a function of spot price, and then developing an approximate replicating portfolio employing standard instruments including bonds, forwards, call and put options.

II. PROBLEM FORMULATION AND ANALYSIS

Consider an LSE who is obligated to serve an uncertain electricity demand q at the fixed price r .³ Assume that the LSE procures electricity, that it needs to serve its customers, from the wholesale market at spot price p . We consider a single-period problem where hedging instruments are purchased at time 0 and all payoffs are received at time 1. The hedging portfolio has an overall payoff structure $x(p)$, which depends on the realization of the spot price p at time 1. Note that our hedging portfolio may include money market accounts, letting the LSE borrow money to finance hedging instruments.

Let $y(p, q)$ be the LSE's profit from serving the load at the fixed rate r at time 1. Then, the total profit $Y(p, q, x(p))$ after receiving payoffs from the contracts in the hedging portfolio is given by

$$Y(p, q, x(p)) = y(p, q) + x(p)$$

where

$$y(p, q) = (r - p)q.$$

Assume that the LSE's preference is characterized by a concave utility function U defined over the total profit $Y(\cdot)$ at time 1. LSE's beliefs on the realization of spot price p and load q are characterized by a joint probability function $f(p, q)$ for positive p and q , which is defined on the probability measure P . On the other hand, let Q be a risk-neutral probability measure based on which the hedging instruments are priced in the market⁴, and $g(p)$ be the probability density function of p under Q .⁵ The LSE's problem can then be formulated as follows:

$$\max_{x(p)} E[U[Y(p, q, x(p))]]$$

$$s.t. E^Q[x(p)] = 0$$

³In fact, most of US states which opened their retail markets into competition have frozen their retail electricity prices.

⁴In mathematical finance, a risk-neutral measure, equivalent martingale measure, or Q-measure is a probability measure that results when one assumes that the current value of all financial assets is equal to the expected value of the future payoff of the asset discounted at the risk-free rate

⁵Note that this probability measure may not be unique since the electricity market is incomplete, however, we ignore this issue.

where $E[\cdot]$ and $E^Q[\cdot]$ denote expectations under the probability measure P and Q , respectively. The constraint in the above formulation means that the manufacturing cost of the portfolio is zero, because contract prices are set to the expected value of discounted payoff under the risk-neutral probability measure. This zero-cost constraint implies that purchasing derivative contracts may be financed from selling other derivative contracts or from the money market accounts. In other words, under the assumption that there are no limits on the possible amount of instruments to be purchased and money to be borrowed, our model finds a portfolio from which the LSE obtains the maximum expected utility over total profit.

An alternative criterion for optimal hedging is to maximize expected return subject to a zero cost constraint as above and a constraint on the “value at risk (VaR) which is defined as a value such that the probability of realized total net profit (from energy sales and portfolio payoff) below VaR does not exceed a specified value α ”. In [4] we show, however, that the VaR constrained portfolio can be approximated by a portfolio that maximizes a Mean-Variance utility function of the form:

$$E\{U(Y)\} = E\{Y\} - \gamma VAR(Y)$$

with an appropriate choice of the risk aversion coefficient γ .

As an illustrative example we employ the above Mean-Variance criterion and assume that spot prices and demand volume uncertainty can be represented by a bivariate normal distribution on $\{\log p, \log q\}$. Figure 1 below shows the shape of the optimal hedging payoff function $x^*(p)$ and the replicating portfolio consisting of a bond, an “at the money” forward contract (i.e., a forward contract with a settlement price set to the spot price at contracting time) and an assortment of put and call options with different strike prices. We also show how the replicating portfolio approximates the continuous optimal payoff for different levels of strike price granularity ΔK . The staggered strike prices serve as a means for tracking volume fluctuations that are positively correlated with wholesale prices. As the price increases, more call options are “in the money” thus providing a hedge for a larger quantity. Likewise on the downside as price drops more put options are in the money providing a guaranteed sales price for more excess quantity.

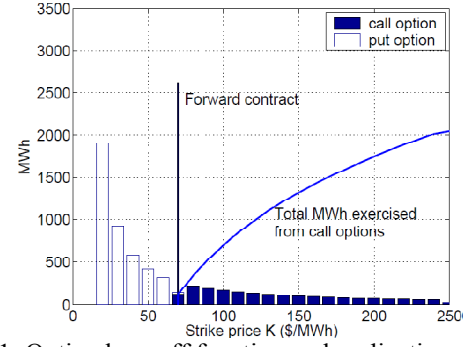
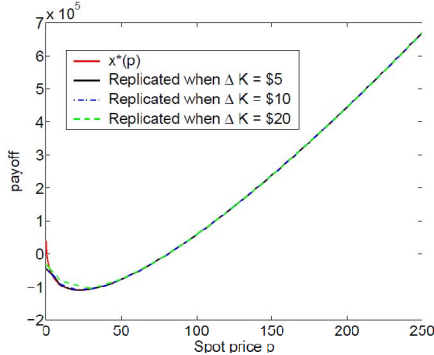


Figure 1: Optimal payoff function and replicating portfolio

Figure 2 shows the total profit distribution without hedging, with price hedging only and with the proposed price and quantity hedging strategy.

To examine the effect of contracting time within the hedging period we must specify the dynamics of price and quantity which we assume to be represented by correlated mean reverting random processes. For illustrative purposes we use the following model which was employed in [6] and calibrated to NordPool data. The model describes price and quantity dynamics with dB_t^1 and dB_t^2 denoting independent brownian motion processes.

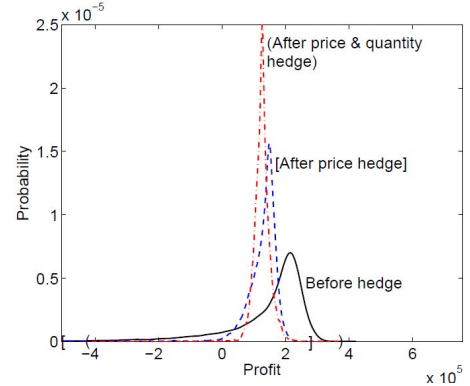


Figure 2: Profit distribution before and after hedging

$$dp_t / p_t = e^{-\psi(T-t)} \sigma dB_t^1$$

$$dq_t / q_t = \phi \sigma_L dB_t^1 + \sqrt{1 - \phi^2} \sigma_L dB_t^2.$$

Figure 3 illustrates the standard deviation of total profit as function of contracting time for various levels of the parameter ψ , representing spot price volatility. It shows that early hedging is a wise strategy. While the optimum contracting time is around midpoint of the hedging period, little is gained by waiting and much can be lost by waiting too long.

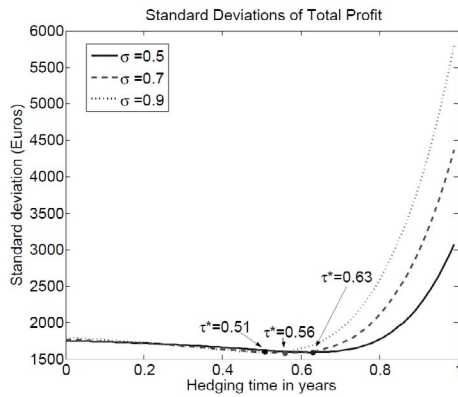


Figure 3: Profit variance vs. contracting time.

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