

# Optimal Transmission Switching with Contingency Analysis

Kory W. Hedman, *Student Member, IEEE*, Richard P. O'Neill, Emily Bartholomew Fisher, *Student Member, IEEE*, and Shmuel S. Oren, *Fellow, IEEE*

**Abstract**--In this paper, we continue to analyze optimal dispatch of generation and transmission topology to meet load as a mixed integer program (MIP) with binary variables representing the state of the transmission element (line or transformer). Previous research showed a 25% savings by dispatching the IEEE 118 bus test case with a DCOF. This paper is an extension of that work by examining the effects of transmission switching with an N-1 DCOF on the IEEE 118 bus and the IEEE 73 bus test case, also known as the RTS 96 system. We demonstrate that these networks can be operated to satisfy N-1 standards while cutting costs by incorporating transmission switching into the dispatch. In some cases, the percent savings from transmission switching was higher with an N-1 DCOF formulation than with a DCOF formulation. We also analyze both IEEE test cases at varying load levels.

**Index Terms**—Integer programming, Power generation dispatch, Power system economics, Power transmission control, Power transmission economics, Power system reliability

## I. NOMENCLATURE

### Indices

$n, m$ : nodes  
 $k$ : transmission element (line or transformer)  
 $g$ : generator  
 $d$ : load  
 $c$ : T-1 or G-1 contingency

### Variables

$\theta_n$ : voltage angle at node  $n$   
 $\theta_{nc}$ : voltage angle at node  $n$  for contingency  $c$   
 $P_{mkn}$ : real power flow from node  $m$  to  $n$  for transmission element  $k$   
 $P_{ng}$ : real power supply from generator  $g$  at node  $n$   
 $P_{nd}$ : real power load at node  $n$   
 $P_{mkn}$ : real power flow from node  $m$  to  $n$  for transmission element  $k$  for contingency  $c$   
 $P_{ngc}$ : real power supply from generator  $g$  at node  $n$  for contingency  $c$   
 $z_k$ : binary variable for transmission element  $k$  (0 open, 1 closed)  
 $TC_J$ : total system cost with  $J$  opened transmission elements

### Parameters

$\theta_n^{max}, \theta_n^{min}$ : max and min voltage angle at node  $n$   
 $P_k^{max}, P_k^{min}$ : max and min capacity of transmission element  $k$   
 $P_g^{max}, P_g^{min}$ : max and min capacity of generator  $g$   
 $P_{kc}^{min}, P_{kc}^{max}$ : max and min transmission element  $k$  emergency rating  
 $c_{ng}$ : cost of production from generator  $g$   
 $B_k$ : electrical susceptance of transmission element  $k$   
 $NI_c$ : binary parameter that is 0 when the element is the contingency and 1 otherwise  
 $T$ : number of transmission elements  
 $G$ : number of generators  
 $J$ : number of open transmission elements  
 $K$ : set of transmission elements allowed to be switched

## II. INTRODUCTION

Transmission elements (lines or transformers) are traditionally treated as fixed assets in the network, except during times of outages. This traditional view does not describe them as assets that operators have the ability to control. However, it is acknowledged, both formally and informally, that system operators can, and do, change transmission elements' state thereby changing the topology of the network. This is typically performed by an operator in order to improve voltage profiles or increase transfer capacity.<sup>1</sup> These decisions are made under a set of prescribed rules by the operator, rather than included in the optimization formulation. The concept of transmission dispatch was introduced by O'Neill et al. [1] in a market context, in which the dynamic operation and compensation of transmission elements are examined.

This paper is one in a series (O'Neill et al. [1], Fisher et al. [2], Hedman et al. [3], and Hedman et al. [4]). [1] introduced the concept of a dispatchable network. [2] provided the formulation for transmission switching, applied it to the IEEE 118 bus system, and discussed the effects on varying load profiles and the practical implications of transmission switching.

[3] applied transmission switching to the IEEE 118 bus test case as well and discussed the financial impacts that transmission switching can have on market participants, the added uncertainty as a result of transmission switching, and the policy implications. [3] discusses the implications of transmission switching with regards to revenue adequacy of financial transmission rights. Even today, revenue adequacy is not guaranteed for FTRs [5]. Empirical evidence suggests that, as long as revenue adequacy is maintained for the static network, there will be revenue adequacy for the transmission switching solution that produces a higher social surplus. However, a formal proof of revenue adequacy with transmission switching has yet to be developed. Even if there is revenue inadequacy, since total surplus is guaranteed not to decrease with transmission switching, there is the possibility for Pareto improvements for all market participants.

[4] uses the same DCOF transmission switching formulation as presented in [2] [3] and applies it to larger problems: the ISONE network and an equivalent representation of the CAISO network.<sup>2</sup> The results from [4] show that transmission switching may be beneficial with a large scale network as well as obtained within a reasonable timeframe with the use of simple heuristic techniques. [4] also

<sup>1</sup> Personal communication with Andy Ott, Vice President PJM.

<sup>2</sup> A longer version of [4] can be found online [7].

presents a discussion on LaGrangian Relaxation (LR) versus Mixed Integer Programming (MIP) and how MIP is already being used in the energy industry to save millions as compared to LR [6]. [2], [3], and [4] all used a DC Optimal Power Flow (DCOPF) to solve the power flow problem.

Though these previous papers do not address reliability, we are not suggesting dispatching transmission at the expense of reliable network operations. The purpose is to examine the potential for automating actions operators currently take, such as implementing special protection schemes (SPSs), and improving network operation by making use of controllable components. Transmission elements that are open in the optimal dispatch of a network may be available to be switched back into the system as needed, as in PJM’s SPSs. In cases where this may not be possible, transmission switching can be conducted in conjunction with contingency analysis in order to maintain reliability levels while taking advantage of improved topology, as is shown in this paper. However reliability is maintained, transmission dispatch is not by definition incompatible with reliable operation of the grid.

The objective of this paper is to demonstrate that transmission switching may provide benefits by extending the analysis to incorporate N-1 contingency analysis with the DCOPF transmission switching formulation.<sup>3</sup> This work applies the technique to the IEEE 118 bus test case as well as the IEEE 73 bus test case, also known as RTS 96 [9] [10]. In theory, this problem is NP hard so optimality is difficult to achieve. In a practical setting, proving optimality is less important than improving the solution; the objective is to find the best solution within the available timeframe. In practice, operators typically have two hours to determine dispatch by running the OPF. The results in this paper do not reflect optimal solutions; rather they reflect our best found feasible solutions.

Earlier work has shown that transmission switching provides flexibility to the grid and may be used as a control method for issues including voltage stability, line overloading [11], loss or cost reduction [12], system security [13], or a combination of these [14] [15] [16] [17]. This work investigates how transmission switching can increase economic efficiency while maintaining an N-1 secure network.

The paper is organized as follows. Section III provides a discussion on transmission switching and transmission planning as well as a discussion on the effects of transmission switching on various OPFs. Section IV presents the model formulation for the transmission switching problem. Section V provides a network overview of the IEEE 118 bus test case along with the results. Section VI presents a network overview and the results and analysis for the RTS 96 system. Section VII provides a short discussion on possible future work and section VIII concludes this paper.

### III. DISCUSSION OF TRANSMISSION SWITCHING

#### A. Discussion of Transmission Switching and Transmission Planning

The objective of transmission switching is to maximize the market surplus. Here without price-responsive demand, the objective is to cut generation costs. Transmission switching can provide savings for a variety of reasons. For example, as load patterns change the optimal topology can change. Transmission planning is usually very granular and does not consider all potential market realizations.

There are additional reasons why transmission switching is beneficial: uncertainty of future network conditions, suboptimal transmission planning due to not considering transmission switching in the planning process, etc. The basis for optimality of transmission planning is the aggregate of benefits from the transmission element over the lifecycle of the element; there is no guarantee that the element is beneficial for all periods within its lifecycle. Simple optimization theory implies that an optimal solution to an aggregate problem need not be the optimal solution for the individual periods within this problem. Consequently, transmission switching can be beneficial even with well planned networks.

#### B. Effects of Transmission Switching on Various OPFs

Transmission switching using MIP will not be implemented if it is not shown to be beneficial under an ACOF or SCOPF model within a reasonable timeframe. This paper only presents results from an N-1 DCOPF transmission switching model; paradoxically, savings from implementing transmission switching may actually provide additional savings with more complicated models like ACOF or SCOPF. With more constraints in the original problem and a smaller feasible region, there exists more opportunity to improve the solution. For example, the RTS 96 system shows little savings from transmission switching with a DCOPF but does experience savings under the N-1 DCOPF model, as the results show in this paper. As shown by Figure 1, the most basic dispatch model that has the largest feasible region is an unconstrained economic dispatch followed by DCOPF, N-1 DCOPF, ACOF, and N-1 ACOF as these later models have more constraints that are not redundant.

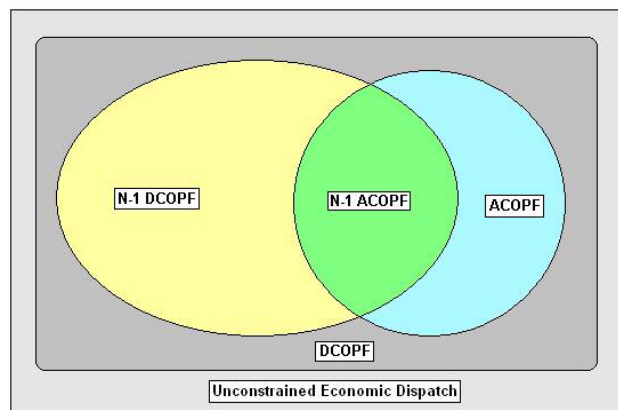


Figure 1. OPF Feasible Region

<sup>3</sup> A longer version of this paper is available online, see [8]

The unconstrained economic dispatch solution is a lower bound to all of these OPFs. In other words, the transmission switching solution can never improve upon the unconstrained economic dispatch. Transmission switching can potentially expand the feasible region of the more heavily constrained problems; consequently, the savings from optimal transmission switching may be greater when used with more constrained OPFs. Likewise, there may be additional costs to consider in more complicated models like startup costs when generation unit commitment is incorporated into the formulation. Optimal transmission switching could potentially provide savings by obviating the need for a generator to start up thereby saving the startup cost. Conversely, one can create a simple example where there are savings from transmission switching for a DCOPF formulation but there are no savings with an N-1 DCOPF formulation.

#### IV. MODEL FORMULATION

##### A. MIP Transmission Switching DCOPF Model

The DCOPF transmission switching formulation has been previously presented in [2], [3], and [4]. The objective is to minimize generation cost subject to physical constraints of the system and Kirchhoff's laws governing power flow. The chosen min and max bus angle values are  $\pm 0.6$  radians.  $M_k$ , listed in (4a) and (4b), is referred to as the "big M" value.  $z_k$  is the binary variable representing the state of the transmission element; a value of one reflects a committed or closed element and a value of zero reflects an uncommitted or open element. When the binary variable  $z_k$  is one, the value of  $M_k$  does not matter; when the binary variable is zero, the value of  $M_k$  is used to ensure that (4a) and (4b) are satisfied regardless of the difference in the bus angles.  $P_{nmk}$  is zero when  $z_k$  is zero so  $M_k$  must be a large number greater than or equal to  $B_k(\theta_n^{\max} - \theta_m^{\min})$ . It is computationally favorable to have  $M_k$  be as small as possible; thus, we set it equal to  $B_k(\theta_n^{\max} - \theta_m^{\min})$ . Without this adjustment to the power flow equations, the buses that are connected to this opened transmission element would be forced to have the same bus angle. Forcing the buses' angles to be the same is incorrect as the element is no longer present. For the situation where there are two parallel transmission lines, if the program removes one line, the other would be forced to have a zero power flow value without this adjustment to the power flow equations. Only with this adjustment does the DCOPF provide the solution corresponding to the case when the transmission element is not present in the network. Injections into a bus are positive (generator supply, power flow into a bus) and withdrawals are negative (load, power flow out of a bus).

$$\text{Minimize: } TC_J = \sum_g c_{ng} P_{ng}$$

s.t.:

$$(1) \theta_n^{\min} \leq \theta_n \leq \theta_n^{\max} \quad \forall n$$

$$(2a) P_g^{\min} \leq P_{ng} \leq P_g^{\max} \quad \forall g$$

$$(2b) P_k^{\min} z_k \leq P_{nmk} \leq P_k^{\max} z_k \quad \forall k$$

$$(3) \sum_{\forall k|l=n} P_{ijk} - \sum_{\forall k|j=n} P_{ijk} + \sum_{\forall g|s=n} P_{sg} + P_{nd} = 0 \quad \forall n$$

$$(4a) B_k(\theta_n - \theta_m) - P_{nmk} + (1 - z_k)M_k \geq 0 \quad \forall k$$

$$(4b) B_k(\theta_n - \theta_m) - P_{nmk} - (1 - z_k)M_k \leq 0 \quad \forall k$$

$$(5) z_k = \{0,1\}, k \in K \quad \forall k$$

Equation (5) is used to allow only a specified set of transmission elements to be switched. For our studies we have  $K = \{k \mid \sum_k (1 - z_k) = J\}$  but this can be adjusted to eliminate elements that can never be opened due to reliability standards. Determining what elements are not candidates for transmission switching would decrease the solution time. We are not advocating introducing (5) to solve practical problems; this constraint is only used to gain understanding about the effects of changing the network topology for various solutions. To solve the transmission switching problem to optimality, (5) would not be present.

##### B. MIP Transmission Switching N-1 DCOPF

The N-1 formulation contains the same equations and objective as above, but adds constraints representing the loss of any single element in the system: a transmission element or generator. The additional constraints are provided below.

s.t.:

T-1 and G-1 constraints

$$(6) \theta_n^{\min} \leq \theta_{nc} \leq \theta_n^{\max} \quad \forall n, c$$

$$(7) P_{kc}^{\min} z_k N1_c \leq P_{nmkc} \leq P_{kc}^{\max} z_k N1_c \quad \forall k, c$$

T-1 constraints only

$$(8) \sum_{\forall k|l=n} P_{ijk} - \sum_{\forall k|j=n} P_{ijk} + \sum_{\forall g|s=n} P_{sg} + P_{nd} = 0 \quad \forall n, \text{T-1 } c$$

$$(9a) B_k(\theta_{nc} - \theta_{mc}) - P_{nmkc} + (2 - z_k - N1_c)M_k \geq 0 \quad \forall k, \text{T-1 } c$$

$$(9b) B_k(\theta_{nc} - \theta_{mc}) - P_{nmkc} - (2 - z_k - N1_c)M_k \leq 0 \quad \forall k, \text{T-1 } c$$

G-1 constraints only

$$(10) P_g^{\min} N1_c \leq P_{ngc} \leq P_g^{\max} N1_c \quad \forall g, \text{G-1 } c$$

$$(11) \sum_{\forall k|l=n} P_{ijk} - \sum_{\forall k|j=n} P_{ijk} + \sum_{\forall g|s=n} P_{sgc} + P_{nd} = 0 \quad \forall n, \text{G-1 } c$$

$$(12a) B_k(\theta_{nc} - \theta_{mc}) - P_{nmkc} + (1 - z_k)M_k \geq 0 \quad \forall k, \text{G-1 } c$$

$$(12b) B_k(\theta_{nc} - \theta_{mc}) - P_{nmkc} - (1 - z_k)M_k \leq 0 \quad \forall k, \text{G-1 } c$$

This DCOPF model does not incorporate generation unit commitment or other generator characteristics such as ramp rates. When there is a generator outage, the system is allowed to be redispatched in order to meet load during this contingency; a generator can be redispatched at any level while satisfying (10). The associated cost of this redispatch is not included in the objective function because the redispatch occurs in real time, where as this model determines the short-

term forward dispatch of the system. Because the probability of an outage is low, we are concerned with feasibility of surviving a contingency, not the cost. Thus, there is a new generation dispatch variable for each G-1 contingency. This variable is represented by  $P_{ngc}$ . The T-1 contingencies, however, are satisfied with pre-contingency power variable  $P_{ng}$  because generators are not redispatched when there is a transmission contingency.

C. LMP Formulation and Pricing

Now that there are constraints reflecting the contingencies, the LMP for every bus is not only based on the shadow price, or dual variable, of the single node balance equation as shown by (3) but the LMP is influenced by the contingencies' node balance equations (8) and (11) as well. All of the shadow prices of the node balance constraints sum to equal the LMP. Let  $\lambda_n$  represent the shadow price of (3) for bus  $n$  and  $\lambda_{nc}$  be the shadow price for bus  $n$  representing contingency  $c$ . The LMP is represented by (13).

$$(13) LMP_n = \lambda_n + \sum_c (\lambda_{nc}) \quad \forall n$$

We assume a nodal pricing system in this study. Generators have linear costs and the generation cost is the total network production cost. Generation revenue is the sum of all individual generator revenues, which is calculated as the bus LMP times the dispatch. Generation rent, or short-term generation profit, is the generation revenue minus generation cost. Congestion rent is the sum of all transmission elements' individual congestion rent, which is calculated as the difference in LMP across the transmission element times the power flow. Load payment is defined as the sum of all load times its LMP.

D. Software Description

The model is written in AMPL, which uses CPLEX. Before the problem is sent to CPLEX, AMPL performs a presolve routine that eliminates any redundant or unnecessary variables and constraints, fixes certain variables, eliminates slack constraints, and it may adjust the objective to compensate for fixing a variable. The problem is then solved by CPLEX, which uses a combination of cut, branch and bound techniques to solve the MIP. For these models, presolve does not modify the objective but many of the slack constraints of (1), (2a), (2b), (6), (7), and (10) are eliminated. The removal of contingency constraints by presolve is expected as some of the constraints are never binding. Some of the binary variables representing the transmission element's status are fixed by presolve; any transmission element that must remain closed to maintain the reliability standards is fixed to one.

V. IEEE 118 BUS TEST CASE – NETWORK OVERVIEW, RESULTS AND ANALYSIS

A. Network Overview

The IEEE118 network data presented in [9] does not include generator cost information. The generator cost information

used in the IEEE 118 network is taken from [18]. Table I lists the network information for the IEEE 118 bus test case while Table II identifies the variables and constraints for both the basic DCOPF as well as the N-1 DCOPF. The generator cost information for this study is relatively low compared to generator costs found in bulk power systems; most generators here have a linear cost that is around \$0.50/MWh with a few expensive generators that are over \$1/MWh. In this paper, we therefore focus on percent savings rather than the dollar value. The average cost of energy for the N-1 DCOPF solution in section V.B. is \$0.735/MWh. If all generator costs were scaled up by a factor of 50 or 100, the average cost of energy would be more typical, and the savings presented in the following sections would be more significant. In order to use a published source, we did not modify the cost information.

More binary variables are eliminated by presolve for the N-1 DCOPF formulation than the DCOPF formulation; there are 177 post-presolve binary variables for the DCOPF whereas there are only 97 for the N-1 DCOPF. This may be both limiting and beneficial for transmission switching. With fewer transmission elements that can be opened, the feasible region is smaller and, therefore, the savings may be reduced. However, with fewer binary variables, the problem is less complex and this helps reduce the computational time. Reducing the computational time is crucial for practical implementation of transmission switching and, in theory, with more advanced OPFs, we may see even fewer transmission elements that can be switched. Likewise, operators could determine which elements cannot be opened in advance in order to simplify the problem.

Table I. IEEE 118 Network Data

	No.	Capacity (MW)			Cost (\$/MWh)	
		Total	Min	Max	Min	Max
Transmission	186	49,720	220	1,100		
Generators	19	5,859	100	805	0.1897	10
Load	99	4,519	2	440		

Table II. IEEE 118 – LP and MIP Variables and Constraints

	DCOPF		N-1 DCOPF	
	LP	MIP	LP	MIP
Total Variables:	323	509	63k	63k
Binary Variables:	0	186	0	186
Total Linear Constraints:	628	1000	126k	202k
Total Variables (Post Presolve):	315	492	60k	61k
Binary Variables (Post Presolve):	0	177	0	97
Linear Constraints (Post Presolve):	482	833	98k	137k

The IEEE 118 bus test case information in [9] does not contain emergency ratings for the transmission elements. [18] lists emergency thermal ratings, or rate C, for the transmission elements. We analyzed two different cases: the first case assumes that the emergency thermal rating for transmission elements is 125% of the steady state operating limit, or rate A, and the second case was based on the emergency ratings listed in [18], which is 113.6% of the steady state limit. There are a number of radial transmission elements within the system that are not subject to reliability standards as defined by FERC<sup>4</sup> so these elements are not included in the N-1 contingency list.

<sup>4</sup> ERO Reliability standards adopted by FERC in Order 696 [19] (See standard TPL-002).

Prior to introducing transmission switching, the systems were checked for compliance with N-1 contingency requirements. The first case could not satisfy N-1 standards without modifications; the system could not survive the loss of either of the two largest generators as well as any of three key transmission elements. Once these items were removed from the N-1 contingency list, the system was N-1 compliant. The second case based on the emergency ratings in [18] did not satisfy N-1 as well. Further investigation found that the same transmission elements and generators removed from the first case's N-1 list needed to be removed for this case as well as one additional transmission element and one additional generator. These items were removed from the N-1 contingency list in order to obtain a base case feasible N-1 DCOPF solution. The transmission elements and generators removed from the contingency list for both cases are listed in Table III.

**Table III. Assets Removed from the IEEE 118 N-1 Contingency Lists**

	Non Radial Transmission Elements [Radial Elements]	Generators
Case 1	(82-83), (89-90), (91-92) [(12-117), (14-15), (16-17), (18-19), (29-31), (68-116), (71-73), (85-86), (86-87), (110-112)]	13, 14
Case 2	Same as case 1 plus: (110-111)	Same as case 1 plus: 17

*B. Results and Analysis Case 1*

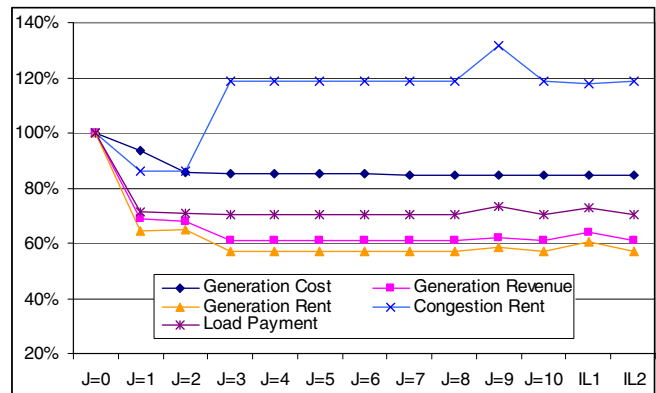
As previously stated, [2] and [3] examined transmission switching with a DCOPF and found significant savings, the best found solution provided a savings of 25% by opening 38 transmission elements. This best found solution was within \$0.0004/hr of the greatest lower bound, which corresponds to an optimality gap of 0.000026%. The base case DCOPF generation cost found in [2] and [3] was \$2,054/hr, generation revenue was \$3,850/hr, generation rent was \$1,795/hr, congestion rent was \$3,907/hr, and load payment was \$7,757/hr.

When incorporating N-1 contingency constraints into the DCOPF, the optimal solution has a cost of \$3,323/hr for the J=0 solution. Recall that J represents the number of transmission elements that are allowed to be opened by transmission switching; thus, J=0, which is an LP, is the base case solution in which no element is allowed to be open. The results presented in Figure 2 correspond to solutions when performing an iterative approach by finding the next best element to open, a technique that does not guarantee an overall optimal transmission switching dispatch but delivers good savings with short solution times. The J=10 solution provides a 15% generation cost savings.

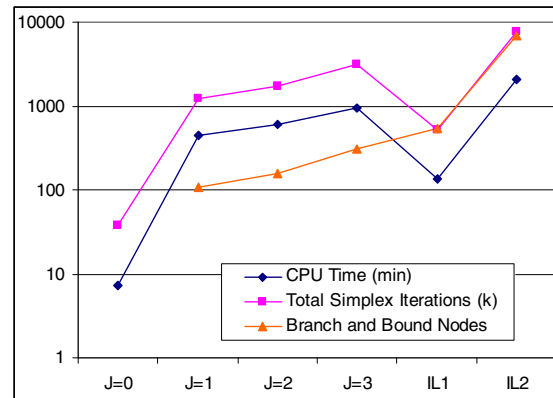
Another solution heuristic, the "intelligent learning" strategy, was employed to arrive at solutions IL1 and IL2. Intelligent learning makes use of familiarity with a particular transmission system. In particular, 20 (for case 1) or 30 (for case 2) specific transmission elements, are eligible for transmission switching. The candidate elements used in the intelligent learning solutions are based on elements that were found from the solutions for J=1 through J=10 presented in this section as well as other elements found when elements were removed for the base DCOPF [3]. Such information would be known once some experience with transmission

switching has been gained. For instance, operators could compile a list of elements that were chosen in the past and have the program focus on these candidate elements, as was done here. Especially with an N-1 contingency analysis, there may be elements that can never be candidates as they are necessary for meeting N-1 reliability standards. Forcing these transmission elements to remain in the network improves the computation time without affecting the objective, as they would never be removed anyway. The results suggest that past information as well as heuristic techniques can be used to obtain good solutions fast.

The computational statistics are displayed in Figure 3. The statistics for solutions obtained by the use of partitioning and heuristics, J={4...10}, are not presented as varying approaches were used to reduce the computation time resulting in the statistics not properly indicating the difficulty of the problems. The cpu time for the intelligent learning solution 1 (IL1) was 134 minutes, which is close to the two hour window operators usually have to solve the scheduling problem; IL1 produced a 15% savings while the J=1 solution took 453 minutes and produced only a 6.3% savings.



**Figure 2. IEEE 118 Results – Case 1**



**Figure 3. Computational Statistics – Case 1**

*C. Results and Analysis Case 2*

Case 2 involved using the emergency ratings, or rate C, listed in [18]. The emergency thermal ratings were 113.6% of the steady state operating limits while case 1 assumed the emergency thermal ratings should be 125% of the steady state operating limit. There was one additional transmission element and generator that were removed from the N-1 contingency list in order for this case's base solution, an N-1

DCOPF without transmission switching, to become feasible. The results are shown in Figure 4; the computational statistics for this case are shown in Figure 5. The N-1 DCOPF J=0 solution has a generation cost of \$3,030/hr. The results are based on taking an iterative approach by finding the next best element to open in the network. Partitioning was used to speed up the process. Computational statistics are not provided for the solutions obtained by partitioning, but performing one complete iteration with partitioning would take about 1 hour if the partitioning was done sequentially. Since the partitioned solutions can run in parallel, the time can be far less. These data resemble the results in the previous section, suggesting that transmission switching is also beneficial with more stringent emergency thermal ratings. Once again, the intelligent learning heuristic solution has a much faster solution time than J=1 and produces a lower-cost dispatch.

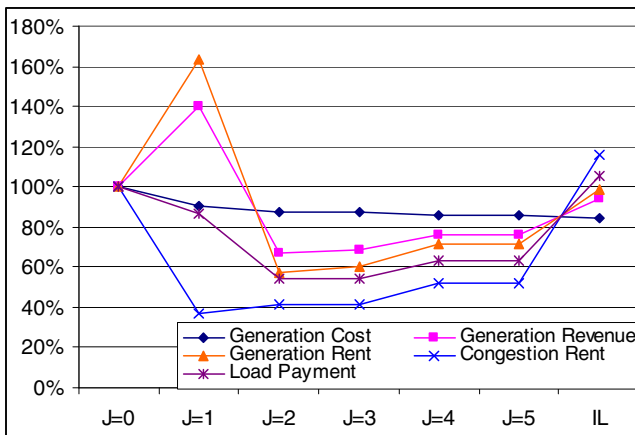


Figure 4. IEEE 118 Results - Case 2

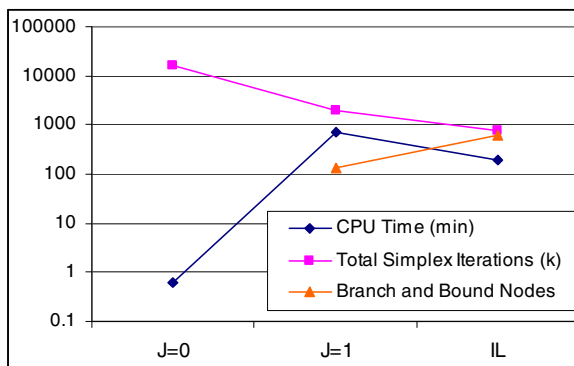


Figure 5. Computational Statistics - Case 2

D. Results and Analysis - Case 1 Load Profile Analysis

This sensitivity study investigates the impact of transmission switching when the load is reduced by 20%. For this load level, the system is N-1 secure except for radial transmission elements. Table III lists the radial transmission elements for the IEEE 118 bus test case.

When the load is reduced by 20%, the DCOPF solution is only \$4/hr greater than the unconstrained economic dispatch solution, leaving little room for improvement from transmission switching [3]. Though the N-1 DCOPF solution is not that close to the unconstrained economic dispatch, the IEEE 118 bus test case does not have a single transmission

element that is thermally constrained at the 80% load level whereas most practical networks would not have such few active thermal constraints at an 80% load level. With over 60,000 thermal and bus angle steady state and contingency constraints, only 10 of them are active (9 thermal contingency constraints, 1 bus angle contingency constraint) for case 1 when the load is at 80% of the peak. The J=1 solution produced a savings of only 0.1% and, after 19 hours, no feasible solution was found for the J=2 solution and the lower bound was 0.2%.

We also analyzed the IEEE 118 bus test case with the load reduced by 10%. Under this situation, a few transmission elements needed to be removed from the contingency list as well as one generator in order to obtain an N-1 DCOPF feasible solution. These assets are identified in Table IV. All of the radial transmission elements were removed from the N-1 contingency list as well.

Table IV. Assets Removed from N-1 Contingency List - 90% Load

Non Radial Transmission Elements	Generators
(82-83)	14

With a 90% load level, transmission switching achieves similar results to those found for the base load level in the previous sections. This was found to be the case for the DCOPF transmission switching model as discussed in [2] where 17% savings were found from transmission switching when the load was reduced by 10%. With this N-1 DCOPF model, transmission switching provides a 13% savings for the best found solution, as is shown by Figure 4. The generation cost for the J=0 N-1 DCOPF solution is \$1,807/hr. Further savings may be obtained with further investigation.

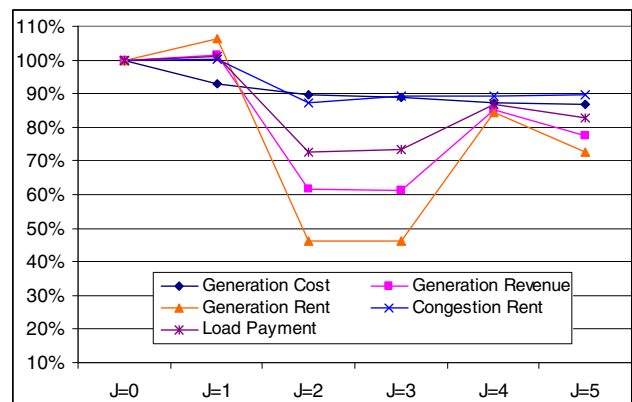


Figure 6. IEEE 118 Results - Case 1 - 90% Load

The solutions presented in Figure 6 were all found by partitioning, except for the J=0 solution. By partitioning the problem into two equal branch and bound trees, the computational time was approximately 2.5 hours with the partitioned problems solved in parallel (the problems were solved at the same time on different machines). Other solutions were partitioned into 20 sets and took approximately 60 to 90 minutes to solve sequentially or at most 10 minutes with parallel computing.

## VI. IEEE 73 RELIABILITY TEST SYSTEM 1996 (RTS 96) – NETWORK OVERVIEW, RESULTS AND ANALYSIS

### A. Network Overview

The IEEE 73 bus network, also known as the three area reliability test system 1996 (RTS 96), was created by a committee of power systems experts to be a standard for reliability testing [10]. The RTS 96 system includes many different configurations and technologies so that it can represent reliability situations found in most electrical systems. It is therefore described in [10] as “a hybrid and atypical system” where load is secure with all elements in service. Nonetheless, it was created as a standard for reliability studies; thus, we chose to include it in our analysis. We found that there were no active thermal transmission or bus angle constraints for the unmodified RTS 96 system for the DCOFF solution, resulting in an unconstrained economic dispatch solution. Consequently, it is common for those performing reliability studies with the RTS 96 system to make modifications. For instance, McCalley et al. [20] use the one area RTS 96 system for a probabilistic security assessment and examine transfers between zones that cause overloading and voltage concerns. They remove a line, shift load, add generation, and change outage rates, among other things, to construct a security constrained area. In particular, the authors adjusted the RTS 96 system by removing line (11-13), shifting 480MW of load from bus 14, 15, 19, and 20 to bus 13, and adding generation capacity at bus 1 (100MW), bus 7 (100MW), bus 15 (100MW, 155MW), and bus 23 (155MW). Buses 14, 15, 19, and 20 had an original total load of 820MW; their new total load is 340MW.

Motto et al. [21] develop an auction market that considers congestion, losses, and other factors. They test their model on the one area RTS 96 system by modifying the rating of a line to create congestion in the network. Specifically these changes include decreasing the thermal capacity of line (14-16) to 350MW. Berizzi et al. [22] use the three area RTS 96 system to illustrate a new ATC calculation that takes into account an N-1 security assessment. They examine maximum power transfer capabilities of main power corridors.

The existence of transmission congestion is central to our study, and thus we created three different test cases based on the modifications inspired by these other studies. The RTS 96 system has three identical zones; the modifications for the first zone are listed below and these same modifications are applied to all zones.

We created three test cases based on the modifications on [20] and [21]. Test case 0 included all of the modifications listed in [20] specified above.<sup>5</sup> Test case 1 includes the modifications in test case 0 and the modifications in [21] specified above. Test case 2 is the same as test case 1; however, it does not include the additional generation capacity referenced in [20].

The network consists of 73 buses, 120 transmission elements (lines and transformers), 111 committed generators with a total capacity of 12,045 MW, and 51 load buses with a total load of 8,547 MW. Table V provides an overview of the components that are modeled within the RTS 96 system. All generators have a minimum operating capacity of 0MW; the generator cost information is an average cost based on the heat rate and fuel cost information presented in [9]. There is seasonal information for the hydro units within the RTS 96 system, all of which are assumed capable of producing at their full capacity. The RTS 96 system includes a yearly load data curve. The results in the following section include the peak hour and solutions corresponding to 70% and 90% of the peak load. Table VI describes the problem size for test case 1; the other test cases are similar in size.

Once again, there are fewer post-presolve binary variables for the N-1 DCOFF than the DCOFF. Certain transmission elements cannot be opened while maintaining N-1 standards so AMPL’s presolve fixes their binary variables’ values to 1.

Table V. RTS 96 System Data

	Capacity (MW)				Cost (\$/MWh)	
	No.	Total	Min	Max	Min	Max
<b>Transmission</b>	120	44,747	175	722		
<b>Generators</b>	111	12,045	12	400	0.00	62.12
<b>Load</b>	51	8,547	53	745		

Table VI. RTS 96 – LP and MIP Variables and Constraints

	DCOFF		N-1 DCOFF	
	LP	MIP	LP	MIP
<b>Total Variables:</b>	304	424	57k	57k
<b>Binary Variables:</b>	0	120	0	120
<b>Total Linear Constraints:</b>	498	738	102k	158k
<b>Total Variables (Post Presolve):</b>	301	421	57k	57k
<b>Binary Variables (Post Presolve):</b>	0	117	0	89
<b>Linear Constraints (Post Presolve):</b>	307	542	72k	75k

### B. Results and Analysis – Peak Load

The results in this section focus on the peak load hour. We tested three modifications of the original RTS 96 system, described in section VI.A. Figures 7 and 8 present the graphical results for test case 1 and test case 2. While test case 0 has limited savings from transmission switching, the other two test cases present an 8% savings from transmission switching while maintaining an N-1 secure network. Further information concerning the results for test case 0 can be found in [8]. The results are not guaranteed to be optimal solutions as they are found by finding the next best transmission element to open. The longest solution took 20 minutes with most taking about 10 minutes. The J=0 N-1 DCOFF solution for case 1 has a generation cost of \$106k/hr and the solution for case 2 has a cost of \$118k/hr.

<sup>5</sup> Modifications in [19] included reducing the total load of several buses. To determine how much load to shift from each individual bus, we calculated each bus’ initial percentage of the original total load among these buses and allocated that bus the same percentage of the new total load. For instance, bus 14 had 23.7% of the 820MW of the original total load and now has 23.7% of the new total load.

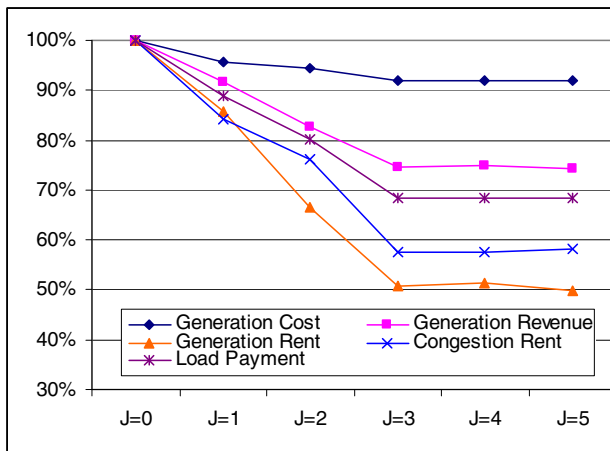


Figure 7. RTS 96 Results for Test Case 1 – Peak Load

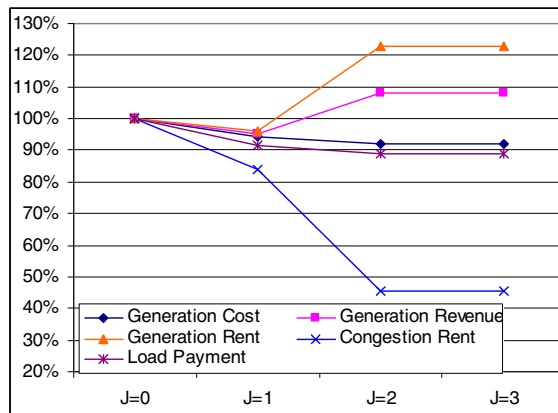


Figure 8. RTS 96 Results for Test Case 2 – Peak Load

C. Results and Analysis – 70% and 90% of Peak Load

The RTS 96 system data in [9] provided load information for the entire year. The previous results reflect the annual peak hour. The mean load level for the peak hours is 70% of the peak load. We assume the peak hours to be hour ending 10:00 and hour ending 21:00 because this is the 12 hour period with the highest mean. The load is then set at 70% of the peak load to analyze the transmission switching savings for lower load levels. With the load at 70% of peak, the savings were found to be about 0.5% for test case 1 and 2, which is not very significant. The RTS 96 system was designed with a robust transmission system that is not often congested as it took many modifications, as described above, to find a result with binding constraints for the N-1 contingency model with peak load. Thus, with lower load levels, these binding constraints are now even fewer. For instance, for test case 1 (test case 2), there are only 40 (30) thermal and bus angle constraints that are active, all of which are contingency constraints, when there are over 40,000 thermal and bus angle constraints. For both test cases 1 and 2, the respective unconstrained economic dispatch solution, which is a lower bound to all OPFs, is only 11% below the N-1 DCOPF solution. With such a close lower bound, there is less possible savings from transmission switching.

Our motivation is to identify the possible savings from transmission switching for IEEE test cases, which then may show the need for further research in this area. Even though

the results suggest that transmission switching does not provide significant savings for lower load levels with an N-1 DCOPF, the same may not be the case for large scale networks. These results arise from IEEE test cases that are lightly congested at lower load levels; thus, these are not generic results. With a DCOPF transmission switching formulation, we researched a 5000 bus ISONE network model and showed significant savings from transmission switching when the load was at 70% or at peak [4]. We have yet to investigate savings from an N-1 DCOPF transmission switching model for ISO networks due to the difficulty in solving the problem.

We continued to study the RTS 96 system with an N-1 DCOPF model but this time the load was set at 90% of peak load. For this study, there are savings from transmission switching unlike the case when the load is at 70% of peak load. The results for test case 1 and test case 2 are presented in Figure 9 and Figure 10. The J=0 N-1 DCOPF solution for case 1 is \$79k/hr while the solution for case 2 is \$86k/hr. With the load reduced by 10%, transmission switching saves up to 4% between these two test cases. All of these solutions were solved by an iterative approach by removing one element at a time so optimality is not guaranteed; therefore, the optimal solution may have more savings than the solutions presented. These solutions were found by partitioning; when the problem was partitioned into two sets, the solution time was up to 20 hours. When the problem was partitioning into many sets, the solution time would be roughly one hour to 90 minutes.

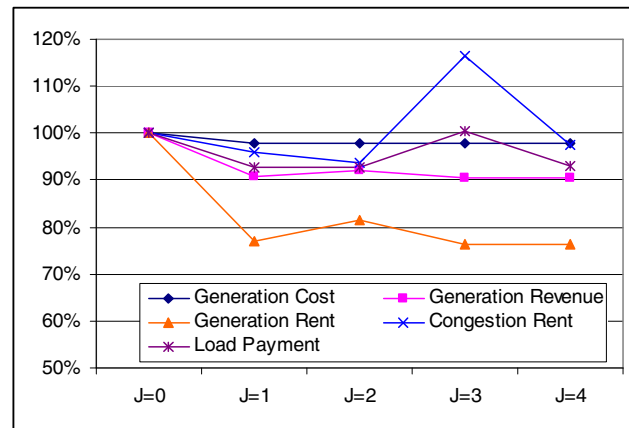


Figure 9. RTS 96 Results for Test Case 1 – 90% Peak Load

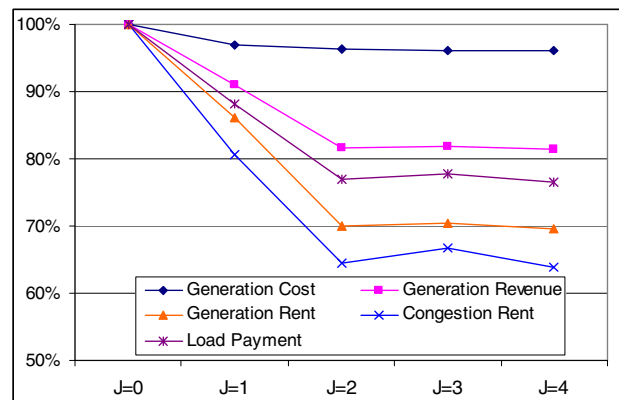


Figure 10. RTS 96 Results for Test Case 2 – 90% Peak Load



## VII. FUTURE WORK

Transmission switching has been shown to provide significant savings when solving the network with a DCOPF. This testing has been performed on the IEEE 118 bus system [2], [3] as well as the ISONE network [4]. This paper has presented that transmission switching also provides savings for an N-1 DCOPF model for the IEEE 118 and the IEEE 73 bus systems. Future research should investigate daily and yearly load patterns to investigate the effects of transmission switching for various load levels. This more expansive analysis should be based on a more complicated model such as an ACOPF or a SCOPF, because lines affect reactive power profiles differently under different loading patterns. Transmission switching may also provide savings by relieving the requirement to start up a generator under certain circumstances, thereby saving the start up costs. A generation unit commitment model should also be built into future work to examine such possibilities. There is also the need to research the impacts from transmission switching regarding real time operations including voltage problems, reactive power, transient stability, etc. This analysis is necessary at varying load levels as well since the capacitive component of a transmission element is predominant during low load levels while the reactive component is predominant at higher load levels.

## VIII. CONCLUSIONS

As computing power and optimization techniques improve, the trillion dollar electric industry looks for ways to cut costs by taking advantage of these improvements. Viewing transmission elements as committable assets is relatively new as such analysis was not possible in the past due to the added complexity to an already challenging problem. This factor is changing and new methods are currently being used to cut costs; for example, PJM is using MIP instead of LaGrangian Relaxation in the unit commitment problem to save millions of dollars [6].

There are concerns with whether transmission switching will be a detriment to reliability and stability. This paper has demonstrated that a network can satisfy N-1 standards while cutting costs by incorporating transmission switching into the dispatch. Significant savings for the IEEE 118 bus test case were obtained due to transmission switching, savings as high as 15% of the generation cost with an N-1 DCOPF model. These savings are not as high as savings found in earlier work that showed a savings of 25% with just a DCOPF model [2]. However, the 15% savings found are still significant. Savings of 8% for the RTS 96 system were obtained with the N-1 DCOPF model whereas there were no savings from transmission switching for the DCOPF model.

These findings suggest that, as one applies a more constrained model, it is possible to obtain more savings from a flexible transmission system. Our work thus far has shown significant savings from transmission switching. If the savings are even half of what we are currently finding, such savings would still be astounding. Such findings suggest that further research is necessary to determine the possible savings from transmission switching for larger networks with more advanced modeling, such as an ACOPF or SCOPF, and

whether transmission switching can be practically implemented.

## REFERENCES

- [1] R. P. O'Neill, R. Baldick, U. Helman, M. H. Rothkopf and W. Stewart, "Dispatchable transmission in RTO markets," *IEEE Trans. Power Syst.*, vol. 20, pp. 171-179, Feb. 2005.
- [2] E. B. Fisher, R. P. O'Neill, M. C. Ferris, "Optimal Transmission Switching," *IEEE Trans Power Syst.*, accepted for publication.
- [3] K. W. Hedman, R. P. O'Neill, E. B. Fisher, S. S. Oren, "Optimal Transmission Switching – Sensitivity Analysis and Extensions," Submitted to *IEEE Trans. Power Syst.*, September 13, 2007.
- [4] K. W. Hedman, R. P. O'Neill, E. B. Fisher, S. S. Oren, "Optimal Transmission Switching Applied to ISO Networks," Submitted to *IEEE Trans. Power Syst.*, January 29, 2008.
- [5] O. Alsac, J. M. Bright, S. Brignone, M. Prais, C. Silva, B. Scott, and N. Vempati, "The rights to fight price volatility." *IEEE Power and Energy Magazine*, vol. 2, no. 4, pp. 47-57, 2004.
- [6] PJM, "News Release: August 14th, 2003: Outage Anniversary," Available: <http://www.pjm.com/contributions/news-releases/2004/20040811-aug-14-outage-anniversary.pdf>. pp. 3, visited October 2007.
- [7] K. W. Hedman, R. P. O'Neill, E. B. Fisher, S. S. Oren, "Optimal Transmission Switching Applied to ISO Networks - Report," To be available online: [www.pserc.wisc.edu](http://www.pserc.wisc.edu)
- [8] K. W. Hedman, R. P. O'Neill, E. B. Fisher, S. S. Oren, "Optimal Transmission Switching with Contingency Analysis – Report," To be available online: [www.pserc.wisc.edu](http://www.pserc.wisc.edu)
- [9] University of Washington, Dept of Electrical Engineering, "Power System Test Case Archive," July 2007. Available: <https://www.ee.washington.edu/research/pstca/index.html>.
- [10] C. Grigg, P. Wong, et al., "The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee," *IEEE Trans. Power Syst.*, vol. 14, Issue 3, pp. 1010-1020, Aug. 1999.
- [11] G. Granelli, M. Montagna, F. Zanellini, P. Bresesti, R. Vailati and M. Innorta, "Optimal network reconfiguration for congestion management by deterministic and genetic algorithms," *Electr. Power Syst. Res.*, vol. 76, pp. 549-556, Apr. 2006.
- [12] G. Schnyder and H. Glavitsch, "Security Enhancement using an Optimal Switching Power Flow," *IEEE Trans. Power Syst.*, vol. 5, pp. 674-681, May 1990.
- [13] G. Schnyder and H. Glavitsch, "Integrated Security Control using an Optimal Power Flow and Switching Concepts," *IEEE Trans. Power Syst.*, vol. 3, pp. 782-790, May 1988.
- [14] R. Bacher and H. Glavitsch, "Network Topology Optimization with Security Constraints," *IEEE Trans. Power Syst.*, vol. 1, pp. 103-111, Nov. 1986.
- [15] J. G. Rolim and L. J. B. Machado, "A study of the use of corrective switching in transmission systems," *IEEE Trans. Power Syst.*, vol. 14, pp. 336-341, Feb. 1999.
- [16] W. Shao and V. Vittal, "Corrective switching algorithm for relieving overloads and voltage violations," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1877-1885, Nov. 2005.
- [17] W. Shao, V. Vittal, "LP-Based OPF for Corrective FACTS Control to Relieve Overloads and Voltage Violations," *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp. 1832-1839, Nov. 2006.
- [18] S. A. Blumsack, "Network Topologies and Transmission Investment Under Electric-Industry Restructuring," PhD dissertation, Engineering and Public Policy, Carnegie Mellon University, Pittsburgh, PA, 2006.
- [19] U.S. Federal Energy Regulatory Commission, "Mandatory Reliability Standards for the Bulk-Power System," Order no. 693, 72 FR 16416, April 4, 2007.
- [20] J. McCalley, S. Asgarpoor, et al., "Probabilistic security assessment for power system operations," PES General Meeting, 2004. vol. 1, pp 212-220, June 2004.
- [21] A. L. Motto, F. D. Galiana, A. J. Conejo, J. M. Arroyo, "Network-constrained multiperiod auction for a pool-based electricity market," *IEEE Trans. Power Syst.*, vol. 17, Issue 3, pp 646-653, Aug. 2002.
- [22] A. Berizzi, M. Merlo, Y. G. Zeng, P. Marannino, P. A. Scarpellini, "Determination of the N-1 security maximum transfer capability through power corridors," PES Winter Meeting, 2000. vol. 3, pp 1739-1744, Jan. 2000.

- [23] R. P. O'Neill, E. B. Fisher, B. F. Hobbs, R. Baldick, "Toward a complete real-time electricity market design," Working Paper.
- [24] P. R. Gribik, D. Shirmohammadi, J. S. Graves, and J. G. Kritikson, "Transmission Rights and Transmission Expansion," *IEEE Trans. Power Syst.*, vol. 20, Issue 3, pp. 1728-1737, Nov. 2005.
- [25] R. Baldick, "Border flow rights and contracts for differences of difference: Models for transmission property rights." *IEEE Trans. Power Syst.*, vol. 22, issue 4, pp 1495-1506, Nov. 2007.
- [26] W. Shao, V. Vittal, "BIP-Based OPF for Line and Bus-bar Switching to Relieve Overloads and Voltage Violations," Proceedings of the 2006 IEEE Power System Conference and Exposition, Atlanta, GA, October 29 – November 1, 2006.

#### BIOGRAPHY

**Kory W. Hedman** is a Ph.D. candidate at the University of California at Berkeley in the dept of industrial engineering and operations research. He received a MS in electrical engineering, thesis option, in May 2007 and a MS in economics in August 2006, from Iowa State University in Ames, IA. He holds a BS in electrical engineering and a BS in economics from the University of Washington, June 2004 in Seattle, WA. He is currently working with FERC as an intern on dispatchable networks.

**Richard P. O'Neill** has a Ph.D. in operations research and a BS in chemical engineering from the University of Maryland at College Park. Currently, he is the Chief Economic Advisor in the Federal Energy Regulatory Commission (FERC), Washington, D.C. He was previously on the faculty of the Department of Computer Science, Louisiana State University and the Business School at the University of Maryland at College Park.

**Emily Bartholomew Fisher** is a Ph.D. student in energy economics and systems at Johns Hopkins University, Baltimore, MD, and an engineer in the Office of Energy Market Regulation at the FERC. She was formerly with the Lawrence Berkeley National Laboratory, and has an undergraduate degree in electrical engineering from Brown University, Providence, RI.

**Shmuel S. Oren** is Professor of IEOR at the Univ. of California, Berkeley. He is the Berkeley site director of PSERC (the Power System Engineering Research Center). He published numerous articles on aspects of electricity market design and has been a consultant to various private and government organizations including the Brazilian regulatory commission, The Alberta Energy Utility Board, the Public Utilities Commission, the Polish system operator, and to the Public Utilities Commission of Texas where he is currently a Senior Advisor to the Market Oversight Division. He holds a B.Sc. and M.Sc. in Mechanical Engineering and in Materials Engineering from the Technion in Israel and he received an MS. and PhD. in Engineering Economic Systems in 1972 from Stanford. Dr. Oren is a Fellow of INFORMS and of the IEEE.