

# Hedging Quantity Risks with Standard Power Options in a Competitive Wholesale Electricity Market

Yumi Oum,<sup>1</sup> Shmuel Oren,<sup>1</sup> Shijie Deng<sup>2</sup>

<sup>1</sup> *Department of Industrial Engineering and Operations Research,  
University of California, Berkeley, California 94720-1777*

<sup>2</sup> *School of Industrial and Systems Engineering, Georgia Institute  
of Technology, Atlanta, Georgia 30332-0205*

Received 3 March 2005; revised 14 October 2005; accepted 28 February 2006

DOI 10.1002/nav.20184

Published online 28 July 2006 in Wiley InterScience (www.interscience.wiley.com).

**Abstract:** This paper addresses quantity risk in the electricity market and explores several ways of managing such risk. The paper also addresses the hedging problem of a load-serving entity, which provides electricity service at a regulated price in electricity markets with price and quantity risk. Exploiting the correlation between consumption volume and spot price of electricity, an optimal zero-cost hedging function characterized by payoff as a function of spot price is derived. It is then illustrated how such a hedging strategy can be implemented through a portfolio of forward contracts and call and put options. © 2006 Wiley Periodicals, Inc. *Naval Research Logistics* 53: 697–712, 2006

**Keywords:** load-serving entity; quantity risk; volumetric risk; electricity market; option; hedging

## 1. INTRODUCTION

Over the past decade, electricity markets in the United States and worldwide have undergone a major transition. Traditionally, the process of delivering electricity from power plants via transmission and distribution lines to the end-users such as homes and businesses was done by a regulated utility company with a regional monopoly. However, recent deregulation and restructuring of the electricity industry vertically unbundled the generation, transmission, and distribution and introduced competition in generation, wholesale procurement, and, to a limited extent, retail supply of electricity. Electricity is now bought and sold in the wholesale market by numerous market participants such as generators, load-serving entities (LSEs), and marketers at prices set by supply and demand equilibrium. As a consequence of such restructuring, market participants are now exposed to price risk, which has fueled the emergence of risk management practices such as forward contracting and various hedging strategies.

The work described in this paper was coordinated by the Consortium for Electric Reliability Technology Solutions (CERTS) on behalf of the Department of Energy. The second and third authors were also supported by NSF Grants EEC 0119301 and ECS 0134210 and by the Power Systems Engineering Research Center.

*Correspondence to:* S. Oren (oren@ieor.berkeley.edu)

The most evident exposure faced by market participants is price risk, which has been manifested by extremely high volatility in the wholesale power prices. During the summer of 1998, wholesale power prices in the Midwest of the United States surged to a stunning amount of \$7,000 per MWh from the normal price range of \$30–\$60 per MWh, causing the defaults of two power marketers on the east coast. In February 2004, persistent high prices in Texas during an ice storm that lasted 3 days led to the bankruptcy of a retail energy provider that was exposed to spot market prices. In California during the 2000/2001 electricity crisis wholesale spot prices rose sharply and persisted around \$500 per MWh. The devastating economic consequences of that crisis are largely attributed to the fact that the major utilities who were forced to sell power to their customers at low fixed prices set by the regulator were not properly hedged through long-term supply contracts. Such expensive lessons have raised the awareness of market participants to the importance and necessity of risk management practices in the competitive electricity market.

Volumetric risk (or quantity risk), caused by uncertainty in the electricity load, is also an important exposure especially for LSEs who are obligated to serve the varying demand of their customers at fixed regulated prices. Electricity volume directly affects the company's net earnings and, more importantly, the spot price itself. Hence, hedging strategies that

only concern price risks for a fixed amount of volume cannot fully hedge market risks faced by LSEs. Unfortunately, while it is relatively simple to hedge price risks for a specific quantity (e.g., through forward contracts), such hedging becomes difficult when the demand quantity is uncertain, i.e., volumetric risk is involved. When volumetric risks are involved a company should hedge against fluctuations in total cost, i.e., quantity times price, but unfortunately, there are no simple market instruments that would enable such hedging. Furthermore, a common approach of dealing with demand fluctuations for commodities by means of inventories is not possible in electricity markets where the underlying commodity is not storable.

The non-storability of electricity combined with the steeply rising supply function and long lead time for capacity expansion results in strong positive correlation between demand and price. When demand is high, for instance due to a heat wave, the spot prices will be high as well and vice versa. For example, the correlation coefficient between hourly price and load for 2 years from April 1998 in California<sup>1</sup> was 0.539. Li and Flynn [20] also calculated the correlation coefficients between normalized average weekday price and load for 13 markets: for example, 0.70 for Spain, 0.58 for Britain, and 0.53 for Scandinavia. There are some markets where this price and load relationship is weak but in most markets load is the most important factor affecting the price of electricity.

The correlation between load and price amplifies the exposure of an LSE having to serve the varying demand at fixed regulated prices and accentuates the need for volumetric risk hedging. An LSE purchasing a forward contract for a fixed quantity at a fixed price based on the forecasted demand quantity will find that when demand exceeds its forecast and it is underhedged, the spot price will be high and most likely will exceed its regulated sale price, resulting in losses. Likewise, when demand is low below its forecast, the spot price at which the LSE will have to settle its surplus will be low and most likely below its purchase price, again resulting in losses.

Because of the strong causal relationship between electricity consumption and temperature, weather derivatives have been considered an effective means of hedging volumetric risks in the electricity market. The advantage of such practices stems from the liquidity of weather derivatives due to their multiple applications. However, the speculative image of such instruments makes them undesirable for a regulated utility having to justify its risk management practices and the cost associated with such practices (which are passed on to customers) to a regulator. In some

jurisdictions, the regulators (e.g., the California Public Utility Commission (CPUC)), who are motivated by concerns for generation adequacy, require that LSEs hedge their load serving obligations and appropriate reserves with physically covered forward contracts and options for power. That is, the hedges cannot just be settled financially or subject to liquidation damages but must be covered by specific installed or planned generation capacity capable of physical delivery. In California, the CPUC has explicitly ordered phasing out of financial contracts with liquidation damages as means of meeting generation adequacy requirements by 2008 [7, 8].

In this paper we propose an alternative to weather derivatives that involves the use of standard forward electricity contracts and price-based power derivatives. This new approach to volumetric hedging exploits the aforementioned correlation between load and price. Specifically, we address the problem of developing an optimal hedging portfolio consisting of forward and options contracts for a risk-averse LSE when price and volumetric risks are present and correlated. We derive the optimal payoff function that maximizes the expected utility of the LSE's profits and determine the mix of forwards and options that replicate the optimal payoff of a hedging portfolio in a single-period setting. While at present the liquidity of power derivatives is limited, we expect that better understanding of how such instruments can be used (which is the goal of this paper) will increase their utilization and liquidity.

Electricity markets are generally incomplete markets in the sense that not every risk factor can be perfectly hedged by market traded instruments. In particular, the volumetric risks are not traded in electricity markets. Thus, we cannot naively adopt the classical no-arbitrage approach of constructing a replicating portfolio for hedging volumetric risks, and the volumetric risk cannot be eliminated. Since there are a lot of portfolios of existing derivative contracts that can partially hedge a given exposure to volumetric risks, the problem is to select the best one according to some criterion. Our proposed methodology is based on an alternative approach offered by the economic literature for dealing with risks that are not priced in the market, by maximizing the expected utility of economic agents bearing such risks [10, 16, 17].

Our mathematical derivation is based on the utility function representation of the risk preference of an LSE. We derive an optimal payoff function that represents payoff as function of price and with zero expected value. We then show how the optimal payoff function can be synthesized from a portfolio of forward contracts, European call and put options. We then provide an example for an LSE considering two alternative forms of its utility function: (1) constant absolute risk aversion (CARA) and (2) mean-variance utility risk preference, under bivariate normal assumptions on the distribution of quantity and logarithm of price.

<sup>1</sup> During this period, all the regulated utilities in the California market procured electricity from the spot market at the Power Exchange (PX). They were deterred from entering into long-term contracts through direct limitations on contract prices and disincentives due to ex post prudence requirements.

Hedging problems dealing with non-traded quantity risk have been analyzed in the agricultural literature; farmers also face correlated price and quantity uncertainty. But the analogy is imperfect since LSEs have different profit structure, higher price volatility due to non-storability, and positive correlation between price and quantity.<sup>2</sup> Furthermore, storage provides an alternative means for handling quantity risk. Nevertheless, the farmer's problem provides some useful references that are relevant to the LSE's hedging problem. In a pioneering article, McKinnon [21] shows that the correlation between price and quantity is a fundamental feature of this problem and calculated the variance-optimizing hedge ratio of futures contracts. He shows that the optimal forward sale cannot completely eliminate a great deal of uncertainty that was introduced to the farmer's income from output uncertainty. The individual farmer can deal with such output variability by investing in buffer stocks, and McKinnon shows that buffer stocks can reduce part of quantity uncertainty. Such storing is not an economical option to consider for the participants in the electricity market, so LSEs will have to rely more on financial derivatives. Some articles consider farmers who find it infeasible to carry buffer stocks from one period to another [22, 24]. For example, Moschini and Lapan [22] shows that quantity uncertainty provides a rationale for the use of options. They derived exact solutions for hedging decisions on futures and options for a farmer with a CARA utility under multivariate normally distributed price and quantity, assuming that only one option strike price is available.

With CARA utility function and bivariate normal distribution for price and quantity, Brown and Toft [3] derives the optimal payoff that should be acquired by a value-maximizing non-financial production firm facing multiplicative risk of price and quantity. Instead of assuming the existence of certain instruments, they derive the payoff function that the optimal portfolio will have. We use their idea of obtaining the optimal payoff function and solve an LSE's problem under different profit, utility, and probability distributions. Moreover, we extend this approach by replicating the optimal payoff function using available financial contracts. Determining the optimal number of contracts from a set of available options requires solving a difficult optimization problem, even in a single-period setting, due to nonlinearity of the option payoff. In this paper we tackle the problem by first determining a continuous optimal hedging function and then developing a replicating strategy based on a portfolio of standard instruments.

Our result shows that we can construct an optimal hedging portfolio for the LSE that includes forwards and options with

<sup>2</sup> Brown and Toft [3] show that firms with the positive price-quantity correlation should hedge more in most price states to compensate for the increased exposure associated with the positive correlation than firms with the negative price-quantity correlation.

various strike prices. The idea of volumetric hedging using a spectrum of options was also proposed by Chao and Wilson [6] from the perspective of a regulator who could impose such hedging on the LSE as a means of ensuring resource adequacy and market power mitigation.

The remainder of the paper is organized as follows. In Section 2, we provide some background about the electricity market that is relevant to the understanding of volumetric risks and contracts that can be used to manage volumetric risk. We also discuss alternative approaches to volumetric risk management. In Section 3, we explore a way of optimally utilizing European call and put options in mitigating price and volumetric risks together. Section 4 concludes the paper.

## 2. VOLUMETRIC RISKS IN THE ELECTRICITY MARKET

In electricity markets, an LSE is uncertain about how much electricity a customer will use at a certain hour until the customer actually turns a switch on and draws electricity. Furthermore, the LSE is obligated to provide the customer with electricity whenever it turns the switch on. In other words, unlike telephone service, there is no busy signal in electricity supply. Consequently, the electricity demand is uncertain and thus results in volumetric risks.

Uncertainty or unpredictability of a demand quantity is a traditional concern for any commodity, but holding inventory is a good solution to deal with quantity risk for those commodities that can be economically stored. However, electricity is non-storable,<sup>3</sup> which is the most important characteristic that differentiates the electricity market from the money market or other commodities markets. Since electricity needs to be purchased at the same time it is consumed, the traditional method of purchasing an excess quantity of a product when prices are low and holding inventories cannot be used by the firms retailing electricity. Moreover, unlike other commodities, LSEs, which are typically regulated, operate under an obligation to serve and cannot curtail service to their customers (except under special service agreement) or pass through high wholesale prices even when they cannot procure electricity at favorable prices.<sup>4</sup> Consequently, volumetric risks in the electricity market require special handling.

In the next subsection, we discuss why volumetric risks are significant in the electricity market. The following subsection explains financial contracts that can be used to mitigate such risk.

<sup>3</sup> The most efficient way of storing electricity produced is to use the limited pump capacity installed in some hydro storage plants. The efficiency of this method is only around 70% [27]. Therefore, it is generally assumed that electricity is non-storable (at least economically).

<sup>4</sup> In fact, most US states that opened their retail markets to competition have frozen their retail electricity prices.

## 2.1. Why Volumetric Risks Are Significant to LSEs

Electricity demand is highly affected by local weather condition; for example, increased need of air conditioners (or heaters) due to hot (or cold) weather increases electricity demand. As a result, the load process is volatile, having occasional spikes caused by extreme weather condition or special events.

On the other hand, electricity demand is inelastic to price levels. Currently, most electricity users don't have incentives to reduce electricity consumption when spot prices are high because they face guaranteed fixed prices and LSEs have an obligation to meet the demand. This price inelasticity of electricity demand combined with the non-storability of electricity makes sudden spot-price changes more likely than in any other commodity markets.<sup>5</sup> Consequently, electricity spot prices exhibit extraordinarily high volatility compared to financial and commodity markets. For example, the typical volatility of dollar/yen exchange rates is (10–20%), LIBOR rates (10–20%), S&P 500 index (20–30%), NASDAQ (30–50%), natural gas prices (50–100%), while the volatility of electricity is (100–500% and higher) [12].

Because profits are a function of quantity multiplied by the price that is extraordinarily volatile and spiky, small uncertainty in demand volume may become very high uncertainty in LSEs' profits. Furthermore, volumetric risks in the electricity market become severe due to adverse movements of price and volume: the sales volume is small when the profit margin is high, while it is large when the margin is low or even negative. This is due to the price inelasticity of demand and the resulting strong positive correlation between price and demand.

## 2.2. Contracts for Volumetric Risk Management

Due to the non-storability, electricity must be produced exactly at the same time it is consumed, and electricity supply and demand must be balanced on a real-time basis; however, market transactions should occur before the demand and system constraints are fully known. For this reason, electricity spot markets have several settlement processes for physical delivery.

Initial settlement is done in the day-ahead market. The prices and quantities in the day-ahead market are determined by matching offers from generators to bids from LSEs by supply and demand equilibrium, usually for each hour of the next day. As time approaches to the delivery hour and more information is revealed on supply and demand conditions,

<sup>5</sup> Boisvert et al. [2] and Caves et al. [5] support this argument and state that price spikes can be mitigated by introducing voluntary market-based pricing in retail markets. However, regulators have not been persuaded to adopt market-based real time pricing at the retail level [15].

additional settlement processes occur in the day-of, hour-ahead, and ex post markets, at different prices.<sup>6</sup> However, the underlying spot price of electricity derivatives is usually the day-ahead price, because the other markets transacted closer to delivery than the day-ahead market are designed primarily for balancing of real-time supply and demand fluctuations.

To manage risks against volatile spot prices for volatile loads, electricity markets have developed various financial instruments that can be settled in advance before the spot market. In this section, we describe various instruments that can be traded to mitigate volumetric risks: fixed-price fixed-volume contracts, vanilla options, swing options, interruptible service contracts, and weather derivatives.

### 2.2.1. Forward and Futures Contracts (Fixed-Price Fixed-Volume Contracts)

A simple solution to volumetric risks would be to just settle a fixed price agreement in advance for a significant amount of volume. Then, only the remaining amount of demand would be exposed to the volatile spot prices, resulting in reduced volumetric risks. This is what forward contracts do.

A forward contract in the electricity market is an agreement to buy or sell electricity for delivery during a specified period in the future at a price determined in advance when the contract is made. In the electricity forward markets, the products are sold as blocks such as on-peak, off-peak, or super-peak.<sup>7</sup>

Forward contracts are over-the-counter (OTC) products. They need not be standardized; instead, they can be structured in the most convenient way to the counterparties: they could be for the delivery to any location during a certain hour, on-peak or off-peak of a day, week, month, season, or year. Because of their flexibility, forward contracts are more popular than futures and are the most liquid and widely used risk management tool in the electricity market.

Futures contracts are of the same type as forwards, but they are standardized. In 1996, the New York Mercantile Exchange (NYMEX) started to trade electricity futures for

<sup>6</sup> A day-of market is for the delivery of electricity for the rest of the day, and an hour-ahead market is for the next couple of hours. Ex post (or real-time) markets transact for reconciling deviations from the predicted schedules.

<sup>7</sup> On-peak power is the power for peak-load period. In the western regions of the United States, standard on-peak power in the forward market is 6 by 16, which means electricity for the delivery from 6:00 to 22:00, Monday through Saturday, excluding North American Electric Reliability Council (NERC) holidays. On the other hand, in the eastern and central regions, on-peak power is defined as 5 by 16, which means electricity for the 16-hour block 6:00 to 22:00, Monday through Friday, excluding NERC holidays. Super-peak power is the power for the super-peak period. In the western region, the super-peak power is 5 by 8, which is for delivery from 12:00 to 20:00, Monday through Friday. Off-peak power is the power during low-demand period, which is the complement to on-peak.

several regions of North America, followed by other exchanges such as Chicago Board of Trade (CBOT) and the Minnesota Grain Exchange (MGE) in the United States and International Petroleum Exchange (IPE) in London. Unfortunately, for a variety of reasons, after the initial burst of trading activities, markets in the United States lost their interest in electricity futures and turned to forward contracts in OTC markets. As a result, NYMEX, CBOT, and IPE have stopped their trading of electricity futures. Although MGE is still trading them, the trading volume is small.

### 2.2.2. Plain-Vanilla Options

An option in the electricity markets obligates the issuer to reimburse the option holder for any positive difference between the underlying market price and the strike price. Compared to a contract that specifies a fixed quantity, an option has the advantage of reducing quantity risks by enabling an LSE to purchase electricity at the strike price only when it is needed and the spot price exceeds the strike price. In particular, a portfolio of call options with many different strike prices would allow the holder to exercise more options the higher is the spot price, thus obtaining more electricity when the spot price is higher, which usually occurs precisely when its load is greater [6].

The electricity options are diverse in contract terms like products, delivery period, and location. Products could be on-peak, off-peak, or round-the-clock. The delivery period could be a month, a quarter, or a year.

The first category of options consists of calendar year and monthly physical options, which are forward options. The exercise of the December 2004 call option at the end of November allows the holder to receive the specified quantity of electricity (in MWh) during the specified hours (such as on-peak, off-peak, or round-the-clock hours) of December at the strike price. In electricity markets, forward options are not widely traded [11]. The second category of options used in the electricity market is a strip of daily options. These options are specified for a given contract period (year, quarter, month, etc.) and can be exercised daily. For example, the holder of a December 2004 daily call option can issue an advance notice on December 15 to receive a specified volume of electricity on December 16 during the on-peak hours, paying the fixed price per MWh. Last, there are hourly options for financial settlements against hourly spot prices during specified blocks of hours like 1, 4, and 8 hours [14].

### 2.2.3. Swing Options

For swing options, the option holder nominates a total fixed amount to be delivered over the contract period and is also given the right to swing the volume received within a

certain range, with limits on the number of swing right over the contract period. While a vanilla option protects against prices for a fixed volume on each day during the delivery period, a swing option allows the holder to respond to volumetric risks by adjusting the volume exercised. Accordingly, the holder can protect more volume when spot prices are spiky than when spot prices are at a normal level. Swing options are well studied in the literature, for example: [9, 15, 18, 26].

### 2.2.4. Interruptible Service Contracts

Interruptible contracts are made with customers who are willing to have their electricity service interrupted by the LSE under specified conditions. In exchange for the interruption option, the LSE typically offers a lower electricity rate to the customer. In the situations where supply or demand shocks occur, the interruptible contracts allow LSEs to interrupt the counterparty's service at a lower cost than serving them by purchasing power at the high spot price. For literature on such contracts, see [12, 25].

### 2.2.5. Weather Derivatives

Weather derivatives give payouts depending on the realized weather variables; thus, they can be used to hedge volumetric risks for various industries whose supply and demand volume is affected by weather conditions. Since the first transaction by energy companies took place in 1997, the transaction volume in weather derivatives has been expanding rapidly among diverse industries such as agriculture, tourism, beverage, ice cream, and entertainment. In addition, weather derivatives are used by investment firms as independent means of diversifying their risks from the existing financial markets because it is widely perceived that the correlations between weather indices and most financial indices are negligible.

Traditionally, hedging against abnormal weather conditions has been done through insurance contracts. These insurance contracts are typically settled to cover against catastrophic weather conditions such as drought and floods. However, these contracts cannot protect against abnormal but less extreme weather conditions, which could also affect profits. The need for an instrument that can be used to hedge such non-catastrophic weather conditions brought the weather derivatives into the market. Moreover, for weather derivatives, there is no need to provide proofs of financial loss to receive payout unlike insurance contracts; payoffs of weather derivatives are decided by the actual weather readings at a weather station specified in the contract.

Among the various weather derivatives in use based on indices such as precipitation, temperature, and wind speed,

the most commonly traded weather derivatives are Heating Degree Days (HDD) and Cooling Degree Days (CDD) derivatives. The HDD (CDD) index is the sum of positive values of average temperature<sup>8</sup> minus 65° during the contract period, mostly a month or a season. The reason for the popularity of degree-days derivatives is not only the transparency of the data and value, but also the high correlation between electricity demand and degree-days. In response to the increased demand, Chicago Mercantile Exchange (CME) started a standard electronic market place for HDD and CDD futures and options since September 1999 now reaching a more than 30,000 annual trading volume.<sup>9</sup> They are also traded in OTC markets such as LIFFE (London International Financial Futures and Options Exchange) and electronic market places like ICE (intercontinental Exchange).

Suppose an LSE decides to mitigate its volumetric risk associated with serving the uncertain electricity demand in its service area during winter. If the upcoming winter is mild, the electricity demand would be low, leaving the LSE with low revenue. Using the fact that the electricity demand increases as the HDD value increases in the LSE's service area, the LSE could buy an HDD put option with strike 2500 and tick amount<sup>10</sup> \$10, for instance. If the upcoming winter were mild and the HDD were 2000, then the LSE would receive \$5000. However, if the realized HDD were greater than the strike value, 2500, then no payout is made from the contracts. In this way, the LSE would offset its low revenues when the weather is unfavorable.

#### 2.2.6. Power-Weather Cross-commodity Derivatives

Since the price risk and volumetric risk are correlated, weather derivatives that only cover the volumetric risk would not be effective without additional hedging of price risk. For example, LSEs definitely don't like the cases where load is too high at the same time as the wholesale price spikes. But if the wholesale price is not very high, then the high load would be generally favorable to them. Such LSEs can benefit from the power-weather derivatives that would give positive payouts when two conditions are both met, for example, whenever temperature is above 80° at the same time as the spot price of electricity is above \$100. The merit of this kind of double-triggered cross-commodity derivatives is that they are cheaper than the standard weather derivatives. They are available in OTC markets and provide efficient tools for volumetric risk management for LSEs.

<sup>8</sup> Mean of maximum and minimum temperature of the day.

<sup>9</sup> [source: www.cme.com].

<sup>10</sup> The tick amount is the money that the put option would payout for one unit of HDD deviation under the strike price.

### 3. OPTIMALLY HEDGING VOLUMETRIC RISKS USING STANDARD CONTRACTS

While weather derivatives can be used to mitigate volumetric risks because of the high correlation between power demand and weather variables, appropriate use of power derivatives would also help in mitigating volumetric risks due to the correlation between power demand and price. The use of standard electricity instruments may be advantageous when an LSE needs to avoid the speculative stigma of weather derivatives. Regulators may view weather derivative trading as a speculative activity and be reluctant to allow the LSE to pass the cost of such risk management tactics to consumers served at regulated rates. Weather derivatives do nothing to insure supply adequacy, which is a major concern in the power industry. As mentioned earlier, concerns for generation adequacy have motivated regulators to require that LSEs hedge their load serving obligations and reserves with contracts that are covered by physical generation capacity.

In this section, we propose a new approach for managing volumetric risks by constructing a portfolio of standard forward contracts and power derivatives whose underlying is the wholesale electricity spot price. If needed, such instruments can be backed by physical generation capacity or interruptible supply contracts that will assure deliverability of the hedged energy.

Consider an LSE who is obligated to serve an uncertain electricity demand  $q$  at the fixed price  $r$ . Assume that the LSE procures electricity that it needs in order to serve its customers from the wholesale market at spot price  $p$ .

To protect against price risk, the LSE can enter into forward contracts to fix the buying price at the forward price  $F$ . First, the number of forward contracts to be purchased needs to be determined. Suppose that the LSE decides to purchase an amount  $\bar{q}$  of forwards; then, the actual demand would be  $\bar{q} + \Delta q$ . Then, the profit that is at risk is  $\Delta q \cdot (r - p)$ . The LSE would want to protect against the situation where either spot price  $p$  is higher than  $r$  and  $\Delta q > 0$  or  $p$  is less than  $r$  and  $\Delta q < 0$ . Now the second question arises: how to manage this remaining risk?

The LSE's strategy could be buying call options with strike prices that are higher than  $r$  and exercised when  $\Delta q > 0$  and buying put options with strike prices less than  $r$  and exercised when  $\Delta q < 0$ . Of course, prices of the call/put options are not negligible. Then, the relevant decision problem is to determine how many put/call options should the LSE purchase and at what strike prices?

The timing of entering into forward and options contracts is also an important decision, since the forward and options prices change as the time approaches the delivery period, reflecting the changing expectations in the market. Optimizing such timing decisions requires solving an

integrated problem of selecting the optimal hedging portfolio and choosing the optimal timing of purchase. However, the timing problem is out of the scope of this paper.<sup>11</sup> Here we restrict ourselves to a single period model where a hedging portfolio is constructed at time 0 in order to reduce risk from serving retail load at time 1. A single-period model will allow us to see the implications of the optimal hedging strategy.

To deal with this hedging problem, we first derive the overall payoff that the optimal hedging portfolio should have as a function of realized spot price  $p$  and then determine how to span this payoff with forwards and options.

### 3.1. Obtaining the Optimal Payoff Function

#### 3.1.1. Mathematical Formulation

In our single-period setting, hedging instruments are purchased at time 0 and all payoffs are received at time 1. Hedging portfolio has an overall payoff structure  $x(p)$ , which depends on the realization of the spot price  $p$  at time 1. Note that our hedging portfolio may include money market accounts, letting the LSE borrow money to finance hedging instruments. Let  $y(p, q)$  be the LSE's profit from serving the load at the fixed rate  $r$  at time 1. Then, the total profit  $Y(p, q, x(p))$  after receiving payoffs from the contracts in the hedging portfolio is given by

$$Y(p, q, x(p)) = y(p, q) + x(p), \tag{1}$$

where

$$y(p, q) = (r - p)q.$$

The LSE's preference is characterized by a concave utility function  $U$  defined over the total profit  $Y(\cdot)$  at time 1. LSE's beliefs on the realization of spot price  $p$  and load  $q$  are characterized by a joint probability function  $f(p, q)$  for positive  $p$  and  $q$ , which is defined on the probability measure  $P$ . On the other hand, let  $Q$  be a risk-neutral probability measure by which the hedging instruments are priced and  $g(p)$  be the probability density function of  $p$  under  $Q$ . Because the electricity market is incomplete, there may exist infinitely many risk-neutral probability measures. We assume that a specific measure  $Q$  was picked according to some optimal criteria.

We formulate the LSE's problem as

$$\begin{aligned} \max_{x(p)} & E[U(Y(p, q, x(p)))] \\ \text{s.t.} & E^Q[x(p)] = 0, \end{aligned} \tag{2}$$

<sup>11</sup> Related work on this topic can be found in [23], which considers the optimal timing of static hedges using only forward contracts.

where  $E[\cdot]$  and  $E^Q[\cdot]$  denote expectations under the probability measure  $P$  and  $Q$ , respectively. In (2), we require the manufacturing cost<sup>12</sup> of the portfolio to be zero under a constant risk-free rate. This zero-cost constraint implies that purchasing derivative contracts may be financed from selling other derivative contracts or from the money market accounts. In other words, under the assumption that there is no limits on the possible amount of instruments to be purchased and money to be borrowed, our model finds a portfolio from which the LSE obtains the maximum expected utility over total profit.

#### 3.1.2. Optimality Conditions

The Lagrangian function for the above constrained optimization problem is given by

$$\begin{aligned} L(x(p)) &= E[U(Y(p, q, x(p)))] - \lambda E^Q[x(p)] \\ &= \int_{-\infty}^{\infty} E[U(Y)|p]f_p(p)dp - \lambda \int_{-\infty}^{\infty} x(p)g(p)dp \end{aligned}$$

with a Lagrange multiplier  $\lambda$  and the marginal density function  $f_p(p)$  of  $p$  under  $P$ . Differentiating  $L(x(p))$  with respect to  $x(\cdot)$  results in

$$\frac{\partial L}{\partial x(p)} = E \left[ \frac{\partial Y}{\partial x} U'(Y) \middle| p \right] f_p(p) - \lambda g(p) \tag{3}$$

by the Euler equation. Setting (3) to zero and substituting  $\frac{\partial Y}{\partial x} = 1$  from (1) yields the first-order condition for the optimal solution  $x^*(p)$  as follows:

$$E[U'(Y(p, q, x^*(p)))|p] = \lambda^* \frac{g(p)}{f_p(p)}. \tag{4}$$

Here, the value of  $\lambda^*$  should be the one that satisfies the zero-cost constraint (2). If  $g(p) = f_p(p)$  for all  $p$ , then (4) implies that the optimal payoff function makes an expected marginal utility from the variation in  $q$  to be the same for any  $p$ .

#### 3.1.3. CARA Utility

A CARA utility function has an exponential form:  $U(Y) = -\frac{1}{a}e^{-aY}$  where  $a$  is the coefficient of absolute risk aversion. With CARA utility, the optimal payoff function  $x^*(p)$ , which

<sup>12</sup> A derivative price is an expected value of the discounted payoff under the risk-neutral measure  $Q$ .

satisfies (4), is obtained as

$$x^*(p) = \frac{1}{a} \left( \ln \frac{f_p(p)}{g(p)} + \ln E[e^{-ay(p,q)}|p] \right) - \frac{1}{a} \left( E^Q \left[ \ln \frac{f_p(p)}{g(p)} \right] + E^Q[\ln E[e^{-ay(p,q)}|p]] \right). \quad (5)$$

PROOF: We see from the special property  $U'(Y) = -aU(Y)$  of a CARA utility function that the following condition holds:

$$E[U(Y^*)|p] = -\frac{\lambda^*}{a} \frac{g(p)}{f_p(p)},$$

which implies that the utility which is expected at any price level  $p$  is proportional to  $\frac{g(p)}{f_p(p)}$ . Then the optimal condition is reduced to

$$E[e^{-a(y(p,q)+x^*(p))}|p] = \lambda^* \frac{g(p)}{f_p(p)}$$

for an LSE with a CARA utility function. Then,

$$x^*(p) = \frac{1}{a} \ln \left( \frac{1}{\lambda^*} \frac{f_p(p)}{g(p)} E[e^{-ay(p,q)}|p] \right) = \frac{1}{a} \left( -\ln \lambda^* + \ln \frac{f_p(p)}{g(p)} + \ln E[e^{-ay(p,q)}|p] \right). \quad (6)$$

The Lagrange multiplier  $\lambda^*$  in the equation should satisfy the zero-cost constraint (2), which is  $\int_{-\infty}^{\infty} x^*(p)g(p)dp = 0$ . That is,

$$\int_{-\infty}^{\infty} \frac{1}{a} \left( -\ln \lambda^* + \ln \frac{f_p(p)}{g(p)} + \ln E[e^{-ay(p,q)}|p] \right) g(p)dp = 0. \quad (7)$$

Solving (7) for  $\ln \lambda^*$  gives

$$\ln \lambda^* = \int_{-\infty}^{\infty} \left( \ln \frac{f_p(p)}{g(p)} + \ln E[e^{-ay(p,q)}|p] \right) g(p)dp.$$

Substituting this into Eq. (6) gives the optimal solution (5).  $\square$

### 3.1.4. Mean-Variance Approach

The mean-variance approach is to maximize a mean-variance objective function, which is linearly increasing in the mean and decreasing in the variance of the profit:  $E[U(Y)] = E[Y] - \frac{1}{2}a \text{Var}(Y)$ . It follows from  $\text{Var}(Y) = E[Y^2] - E[Y]^2$  that

$$U(Y) \equiv Y - \frac{1}{2}a(Y^2 - E[Y]^2)$$

for the mean-variance objective function in an expected utility form. Then, the optimal solution  $x^*(p)$  that satisfies (4) is obtained as

$$x^*(p) = \frac{1}{a} \left( 1 - \frac{\frac{g(p)}{f_p(p)}}{E^Q \left[ \frac{g(p)}{f_p(p)} \right]} \right) - E[y(p,q)|p] + E^Q[E[y(p,q)|p]] \frac{\frac{g(p)}{f_p(p)}}{E^Q \left[ \frac{g(p)}{f_p(p)} \right]}. \quad (8)$$

PROOF: From  $U'(Y) = 1 - aY$ , the optimal condition (4) is as follows:

$$E[1 - aY^*|p] = \lambda^* \frac{g(p)}{f_p(p)}.$$

Equivalently,

$$f_p(p) - aE[Y^*|p]f_p(p) = \lambda^*g(p). \quad (9)$$

Integrating both sides with respect to  $p$  from  $-\infty$  to  $\infty$ , we obtain  $\lambda^* = 1 - aE[Y^*]$ . Substituting  $\lambda^*$  and  $Y^* = y(p,q) + x^*(p)$  into (9) gives

$$f_p(p) - a(E[y(p,q)|p] + x^*(p))f_p(p) = g(p) - a(E[y(p,q)] + E[x^*(p)])g(p).$$

By rearranging, we obtain

$$x^*(p) = \frac{1}{a} - \frac{1}{a} \frac{g(p)}{f_p(p)} + (E[y(p,q)] + E[x^*(p)]) \times \frac{g(p)}{f_p(p)} - E[y(p,q)|p]. \quad (10)$$

To cancel out  $E[x^*(p)]$  in the right-hand side, we take the expectation under  $Q$  to the both sides to obtain

$$0 = \frac{1}{a} - \frac{1}{a} E^Q \left[ \frac{g(p)}{f_p(p)} \right] + (E[y(p,q)] + E[x^*(p)]) \times E^Q \left[ \frac{g(p)}{f_p(p)} \right] - E^Q[E[y(p,q)|p]], \quad (11)$$



and subtract Eq. (11)  $\times \frac{g(p)/f_p(p)}{E^Q[g(p)/f_p(p)]}$  from Eq. (10). This gives the final formula for the optimal payoff function under mean-variance utility as in (8).  $\square$

Note that when we can assume  $P \equiv Q$  in the electricity market, which was empirically justified in [1, 19] for the Nordic electricity forward market, the optimal payoff function under the mean-variance utility becomes

$$x^*(p) = E[y(p, q)] - E[y(p, q)|p]. \tag{12}$$

The first term  $E[y(p, q)]$  is a constant, and the second term  $E[y(p, q)|p]$  is the expected profit given the value of the spot price. This implies that whatever the spot price is realized, the optimal portfolio is the one that makes the expected total profit for any given price under quantity uncertainty to be the same as the expected profit before hedging. This is because maximizing the mean-variance objective function with our zero-cost constraint and  $P \equiv Q$  is the same as just minimizing a variance of profit after hedging.<sup>13</sup> In fact, given the value of  $p$ , the variance of profit is zero after adding the

optimal payoff in (12). We see that the optimal portfolio can remove only the uncertainty in revenue that is correlated with price.

### 3.1.5. Bivariate Lognormal-Normal Distribution for Price and Load

Suppose the marginal distributions of  $p$  and  $q$  as follows:

$$\text{Under } P : \log p \sim N(m_1, s^2), \quad q \sim N(m, u^2), \\ \text{Corr}(\log p, q) = \rho$$

$$\text{Under } Q : \log p \sim N(m_2, s^2).$$

Then, we can get the explicit functions for the optimal payoff. For the **CARA utility**, the optimal payoff function (5) reduces to

$$x^*(p) = \frac{1}{a}(A_1(p) + A_2(p)), \tag{13}$$

where

$$A_1(p) \equiv \ln \frac{f_p(p)}{g(p)} - E^Q \left[ \ln \frac{f_p(p)}{g(p)} \right] = -\frac{m_2 - m_1}{s^2}(\log p - m_2)$$

$$A_2(p) \equiv \ln E[e^{-ay(p,q)}|p] - E^Q[\ln E[e^{-ay(p,q)}|p]]$$

$$= -ar\rho\frac{u}{s}(\log p - E^Q[\log p]) + a\left(m - \rho\frac{u}{s}m_1\right)(p - E^Q[p]) + a\rho\frac{u}{s}(p \log p - E^Q[p \log p])$$

$$+ \frac{1}{2}a^2(-2r(p - E^Q[p]) + p^2 - E^Q[p^2])u^2(1 - \rho^2)$$

$$= -ar\rho\frac{u}{s}(\log p - m_2) + a\left(m - \rho\frac{u}{s}m_1\right)(p - e^{m_2+\frac{1}{2}s^2}) + a\rho\frac{u}{s}\left(p \log p - (m_2 + s^2)e^{m_2+\frac{1}{2}s^2}\right)$$

$$+ \frac{1}{2}a^2(-2r(p - e^{m_2+\frac{1}{2}s^2}) + p^2 - e^{2m_2+2s^2})u^2(1 - \rho^2)$$

<sup>13</sup> This kind of hedging is also considered in [11]: mean-variance hedging reduces to variance minimization when the pricing measure equals to the physical measure because they consider only forward contracts, which have zero expected value before delivery.

and for the **mean-variance utility**, the optimal payoff function (8) reduces to

$$x^*(p) = \frac{1}{a}(1 - B_1(p)) - B_2(p) + B_3B_1(p), \tag{14}$$

where

$$\begin{aligned}
 B_1(p) &\equiv \frac{\frac{g(p)}{f_p(p)}}{E^Q\left[\frac{g(p)}{f_p(p)}\right]} = \exp\left(\frac{m_2 - m_1}{s^2} \log p + \frac{m_1^2 - m_2^2}{2s^2} - \frac{(m_1 - m_2)^2}{s^2}\right) \\
 &= e^{-\frac{(m_1 - m_2)(m_1 - 3m_2)}{2s^2}} p^{\frac{m_2 - m_1}{s^2}} \\
 B_2(p) &\equiv E[y(p, q)|p] = E[(r - p)q|p] = (r - p) \left(m + \rho \frac{u}{s} (\log p - m_1)\right) \\
 B_3 &\equiv E^Q[E[y(p, q)|p]] \\
 &= (r - E^Q[p]) \left(m - \rho \frac{u}{s} m_1\right) + \rho \frac{u}{s} (r E^Q[\log p] - E^Q[p \log p]) \\
 &= (r - e^{m_2 + \frac{1}{2}s^2}) \left(m - \rho \frac{u}{s} m_1\right) + \rho \frac{u}{s} (r m_2 - (m_2 + s^2)e^{m_2 + \frac{1}{2}s^2}).
 \end{aligned}$$

We have used the following formulas in the calculation.

$$\begin{aligned}
 E^Q[\log p] &= m_2 \\
 E^Q[p] &= e^{m_2 + \frac{1}{2}s^2} \\
 E^Q[p \log p] &= (m_2 + s^2)e^{m_2 + \frac{1}{2}s^2} \\
 E^Q[p^2] &= e^{2m_2 + 2s^2} \\
 \frac{g(p)}{f_p(p)} &= \frac{\frac{1}{ps\sqrt{s\pi}} \exp\left(-\frac{1}{2}\left(\frac{\log p - m_2}{s}\right)^2\right)}{\frac{1}{ps\sqrt{s\pi}} \exp\left(-\frac{1}{2}\left(\frac{\log p - m_1}{s}\right)^2\right)} = \exp\left(\frac{m_2 - m_1}{s^2} \log p + \frac{m_1^2 - m_2^2}{2s^2}\right) \\
 E^Q\left[\frac{g(p)}{f_p(p)}\right] &= \exp\left(\frac{m_2 - m_1}{s^2} m_2 + \frac{m_1^2 - m_2^2}{2s^2} + \frac{(m_2 - m_1)^2}{2s^2}\right) = \exp\left(\frac{(m_1 - m_2)^2}{s^2}\right)
 \end{aligned}$$

We've also used  $q|p \sim N\left(m + \rho \frac{u}{s} (\log p - m_1), u^2(1 - \rho^2)\right)$  to obtain

$$\begin{aligned}
 \ln E[e^{-ay(p,q)}|p] &\equiv \ln E[e^{-a(r-p)q}|p] \\
 &= -a(r - p) \left(m + \rho \frac{u}{s} (\log p - m_1)\right) \\
 &\quad + \frac{1}{2} a^2 (r - p)^2 u^2 (1 - \rho^2).
 \end{aligned}$$

### 3.1.6. Bivariate Lognormal Distribution for Price and Load

Suppose the marginal distributions of  $p$  and  $q$ , on the other hand, follow bivariate lognormal distributions as follows:

$$\begin{aligned}
 \text{Under } P : \quad \log p &\sim N(m_1, s^2), & \log q &\sim N(m_q, u_q^2), \\
 \text{Corr}(\log p, \log q) &= \phi
 \end{aligned}$$

$$\text{Under } Q : \quad \log p \sim N(m_2, s^2).$$

Then, we can get the explicit functions for the optimal payoff for the **mean-variance utility**:

$$x^*(p) = \frac{1}{a} (1 - B_1(p)) - B'_2(p) + B'_3 B_1(p), \quad (15)$$

where

$$\begin{aligned}
 B'_2(p) &\equiv E[y(p, q)|p] = E[(r - p)q|p] \\
 &= (r - p)e^{m_q + \phi \frac{u_q}{s} (\log p - m_1) + \frac{1}{2} u_q^2 (1 - \phi^2)}
 \end{aligned}$$

since  $\log q|p \sim N\left(m_q + \phi \frac{u_q}{s} (\log p - m_1), u_q^2(1 - \phi^2)\right)$ , and

$$\begin{aligned}
 B'_3 &\equiv E^Q[E[y(p, q)|p]] \\
 &= r e^{m_q + \phi \frac{u_q}{s} (m_2 - m_1) + \frac{1}{2} u_q^2 (1 - \phi^2) + \frac{1}{2} \phi^2 \frac{u_q^2}{s^2} s^2} \\
 &\quad - e^{m_2 + m_q + \phi \frac{u_q}{s} (m_2 - m_1) + \frac{1}{2} u_q^2 (1 - \phi^2) + \frac{1}{2} (\phi \frac{u_q}{s} + 1)^2 s^2}. \quad (16)
 \end{aligned}$$

### 3.2. Replicating the Optimal Payoff Function

In the previous section, we've obtained the payoff function  $x^*(p)$  that the optimal portfolio should have. In this section, we construct a portfolio that replicates payoff  $x(p)$ .

In [4], Carr and Madan showed that any twice continuously differentiable function  $x(p)$  can be written in the following form:

$$x(p) = [x(s) - x'(s)s] + x'(s)p + \int_0^s x''(K)(K - p)^+ dK + \int_s^\infty x''(K)(p - K)^+ dK$$

for an arbitrary positive  $s$ . This formula suggests a way of replicating payoff  $x(p)$ . Let  $F$  be the forward price for a delivery at time 1. Evaluating the equation at  $s = F$  and rearranging it gives

$$x(p) = x(F) \cdot 1 + x'(F)(p - F) + \int_0^F x''(K)(K - p)^+ dK + \int_F^\infty x''(K)(p - K)^+ dK. \quad (17)$$

Note that  $1, (p - F), (K - p)^+,$  and  $(p - K)^+$  in the above expression represent payoffs at time 1 of a bond, forward contract, put option, and call option, respectively.

Therefore,

- $x(F)$  units of bonds,
- $x'(F)$  units of forward contracts,
- $x''(K)dK$  units of put options with strike  $K$  for every  $K < F$ , and
- $x''(K)dK$  units of call options with strike  $K$  for every  $K > F$

gives the same payoff as  $x(p)$ .

The above implies that unless the optimal payoff function is linear, the optimal strategy involves purchasing (or selling short) a spectrum of both call and put options with continuum of strike prices. This result proves that LSEs should purchase a portfolio of options to hedge price and quantity risk together. Even if prices go up with increasing loads, more call options with higher strike prices are exercised, having an effect of putting price caps on each incremental load.

In practice, electricity derivatives markets, as any derivatives markets, are incomplete. Consequently, the market does not offer options for the full continuum of strike prices, but typically only a small number of strike prices are offered. Our purpose is to best replicate the optimal payoff function using existing options only. Therefore, we need to decide what amount of options to purchase for each available strike price so that the total payoff from those options is equal or close to the payoff provided by the optimal payoff function.

Let  $K_1, \dots, K_n$  be available strike prices for put options and  $K'_1, \dots, K'_m$  be available strike prices for call options where

$$0 < K_1 < \dots < K_n < F < K'_1 < \dots < K'_m.$$

Consider the following replicating strategy, which consists of

- $x(F)$  units of bonds,
- $x'(F)$  units of forward contracts,
- $\frac{1}{2}(x'(K_{i+1}) - x'(K_{i-1}))$  units of put options with strike prices  $K_i, i = 1, \dots, n,$
- $\frac{1}{2}(x'(K'_{i+1}) - x'(K'_{i-1}))$  units of call options with strike prices  $K'_i, i = 1, \dots, m.$

This strategy was obtained by the following approximations:

$$\begin{aligned} & \int_0^F x''(K)(K - p)^+ dK + \int_F^\infty x''(K)(p - K)^+ dK \\ &= \sum_{i=0}^{n-1} \int_{K_i}^{K_{i+1}} x''(K)(K - p)^+ dK \\ & \quad + \sum_{i=1}^m \int_{K'_i}^{K'_{i+1}} x''(K)(p - K)^+ dK \\ & \approx \sum_{i=0}^{n-1} \int_{\max(p, K_i)}^{\max(p, K_{i+1})} x''(K)dK \cdot \frac{1}{2}\{(K_i - p)^+ + (K_{i+1} - p)^+\} \\ & \quad + \sum_{i=1}^m \int_{\min(p, K'_i)}^{\min(p, K'_{i+1})} x''(K)dK \cdot \frac{1}{2}\{(p - K'_i)^+ + (p - K'_{i+1})^+\} \\ & \approx \sum_{i=0}^{n-1} \int_{K_i}^{K_{i+1}} x''(K)dK \cdot \frac{1}{2}\{(K_i - p)^+ + (K_{i+1} - p)^+\} \\ & \quad + \sum_{i=1}^m \int_{K'_i}^{K'_{i+1}} x''(K)dK \cdot \frac{1}{2}\{(p - K'_i)^+ + (p - K'_{i+1})^+\} \\ &= \sum_{i=1}^n \int_{K_{i-1}}^{K_{i+1}} x''(K)dK \cdot \frac{1}{2}(K_i - p)^+ \\ & \quad + \sum_{i=1}^m \int_{K'_{i-1}}^{K'_{i+1}} x''(K)dK \cdot \frac{1}{2}(p - K'_i)^+. \end{aligned}$$

We see that the error from the replicating strategy is very close to zero if there exist put and call options with strike price  $F$  (i.e.,  $K_n \simeq F \simeq K_1$ ) and if  $p$  is realized very close to one of the strike prices. The error will be smaller if the intervals between strike prices are small, especially for the interval within which there is a high probability that  $p$  will fall.

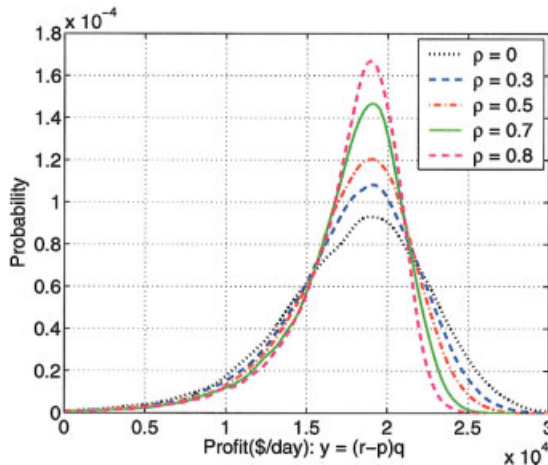
### 3.3. An Example

In this section, we illustrate the method that we derived in the previous sections. We consider the on-peak hours of a single summer day as time 1. Parameters were approximately based on the California Power Exchange data of daily day-ahead average on-peak prices and 1% of the total daily on-peak loads from July to September 1999. Specific parameter values are imposed as follows:

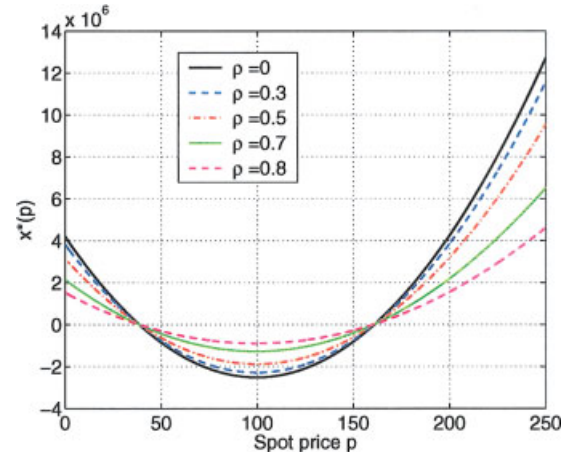
- Price is distributed lognormally with parameters  $m_1 = 3.64$  and  $s = 0.35$  in both the real-world and risk-neutral world:  $\log p \sim N(3.64, 0.35^2)$  in  $P$  and  $Q$ . The expected value of the price  $p$  under this distribution is  $\$40.5/MWh$ .
- The fixed rate  $r = \$100/MWh$  is charged to the customers who are served by the LSE.
- For CARA utility, the risk aversion is  $a = 1.5$ .
- Load is either normally distributed with mean  $m = 300$  and  $u^2 = 30^2$  or lognormally distributed with parameter  $m = 5.77$  and  $u = 0.09$ .

We would like to point out a significant correlation effect in the profit distributions. Figure 1 shows that the profit distributions become quite different as the correlation between load and logarithm of price changes. Considering that the correlation coefficient of our data is 0.7, we observe that the correlation coefficient cannot be ignored in the analysis of profit.

The optimal payoff functions for a CARA utility LSE are drawn in Figure 2 for various correlation coefficients between  $\log p$  and  $q$ . Generally, low profit from high loads for



**Figure 1.** Profit distribution for various correlation coefficients. Generated 50,000 pairs of  $(p, q)$  from a bivariate normal distribution of  $(\log p, q)$  with a various correlation  $\rho$ 's, where  $\log p \sim N(3.64, 0.35^2)$  and  $q \sim N(300, 30^2)$  and plotted estimated probability density functions of the profit using normal kernel (assuming  $r = \$100/MWh$ ). [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]



**Figure 2.** The optimal payoff function for an LSE with CARA utility when price and load follow bivariate lognormal-normal distribution  $\log p \sim N(3.64, 0.35^2)$  and  $q \sim N(300, 30^2)$  with correlation coefficient  $\rho$ . [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]

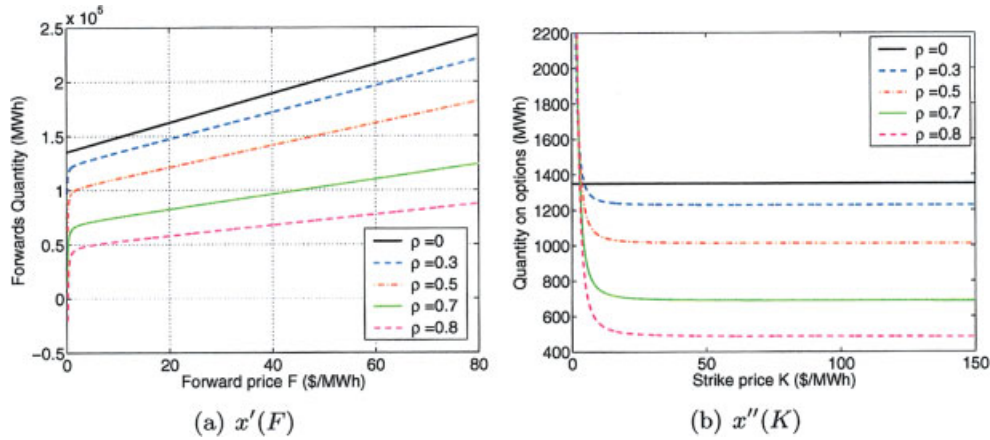
very high spot prices and from low load for very low spot price is compensated with the cases where spot prices and loads are around the expected value. This can be seen from the graph where as the spot price goes away from  $r$ , positive payoff is received from the optimal portfolio while the payoff is negative around  $r$ . We also note that larger payoff can be received when the correlation is smaller. This is because the variance of profit is bigger when the correlation is smaller, as we can see from Figure 1. Therefore, even when the correlation is zero, the optimal payoff function is nonlinear.

Figure 3 illustrates the numbers of contracts to be purchased in order to obtain the payoff  $x^*(p)$  for an LSE with a CARA utility function. We see that the numbers of options contracts are very high relative to the mean volume. This is because we don't restrict the model with constraints such as credit limits. The zero-cost constraint (2) that we only included in our model enables borrowing as much money as needed to finance any number of derivative contracts.

For an LSE with mean-variance utility, the optimal payoff functions are drawn in Figure 4. They show the tendency of mean-variance utility to protect against high price and low quantity. For an illustration of the numbers of contracts to be purchased in order to obtain payoff  $x^*(p)$ , see Figure 5. Note that in our examples the number of options contracts to be purchased in the optimal portfolio is positive for any strike prices. This implies that we borrow money from the bank and purchase a portfolio of options contracts.

Figure 6 compares distribution changes between profit without hedging, profit after price hedge<sup>14</sup> and profit after

<sup>14</sup> Price hedge here means that we add the optimal payoff function obtained under the assumption of no quantity risk. This is in fact equivalent to buying forwards for the average load quantity.



**Figure 3.** The graphs show numbers of forward and options contracts to be purchased in order to replicate the optimal payoff  $x^*(p)$  that is obtained for the LSE with CARA utility. In this example, the forward price is \$40.5/MWh, thus, the optimal portfolio includes forward contracts for  $x'(40.5)$  MWh, put options on  $x''(K)dK$  MWh for  $K < 40.5$  and call options on  $x''(K)dK$  MWh for  $K > 40.5$ . [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]

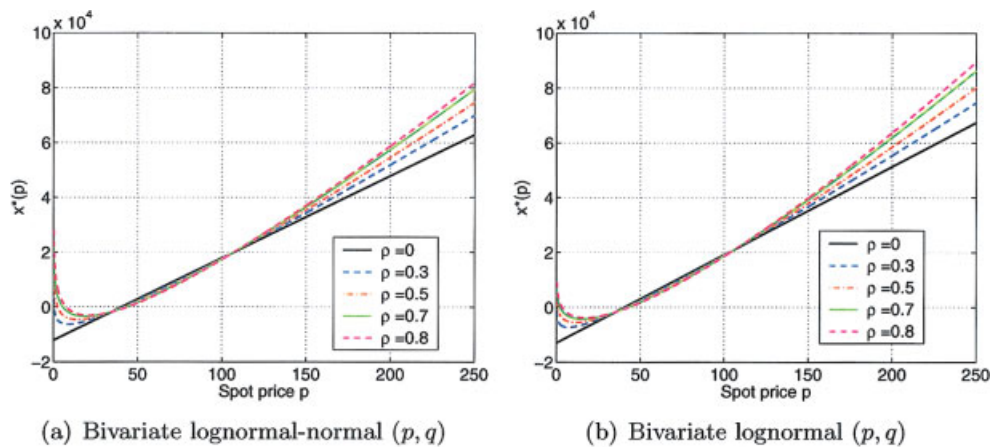
the optimal price and quantity hedge.<sup>15</sup> The graphs shows significant improvements in reducing risks when we hedge price and quantity risk together.

In Figure 7 we explore the sensitivity of the optimal payoff function with respect to the divergence between the risk-neutral distribution and the assumed physical distribution of prices. Specifically, we assume that the joint distribution for quantity and price under both measures  $P$  and  $Q$  is represented by a bivariate lognormal-normal density function with possible differences in the mean logarithmic price, which we vary. The results depend on the utility function used. For CARA utility, the overall payoff is higher than

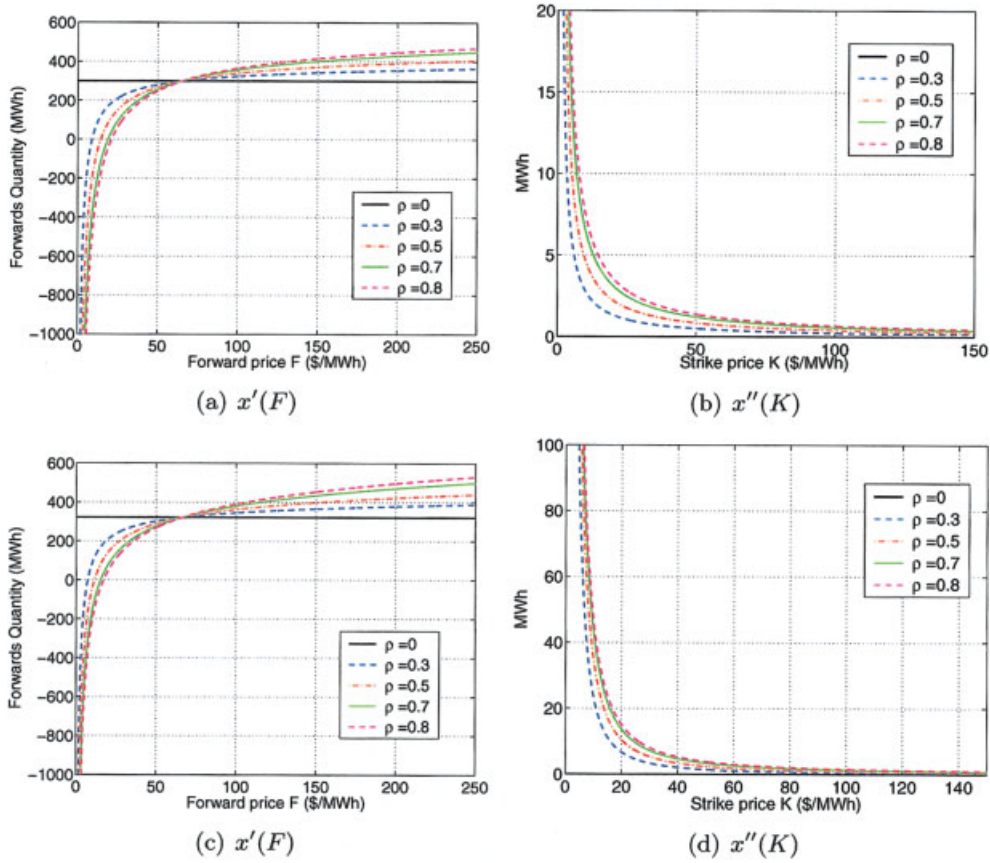
<sup>15</sup> Price and quantity hedge refers to the optimal payoff function that we obtained in this paper.

the optimal payoff if the expected price is higher than the market price, but the difference is not that significant with respect to the payoff changes for differing  $p$ . For the mean-variance case, however, the difference between payoffs for varying mean logarithmic price  $m_2$  is more noticeable.

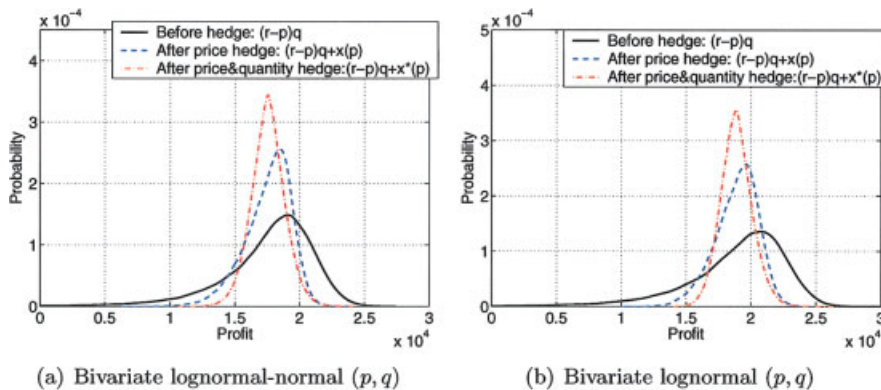
Figure 8 shows how hedging strategies change with risk aversion. Figure 8a displays the optimal payoff functions for CARA utility with different levels of risk aversion. It shows that the payoff function with high risk aversion is more sensitive to the unit change in spot price, indicating that a more risk averse LSE will enter into more active hedging. On the other hand, mean-variance utility shows different aspects. In Figure 8b, as  $a$  gets close to 0.01, the optimal payoff doesn't change much; the mean-variance objective function gives more weight to variance as  $a$  gets bigger, so  $a$  won't



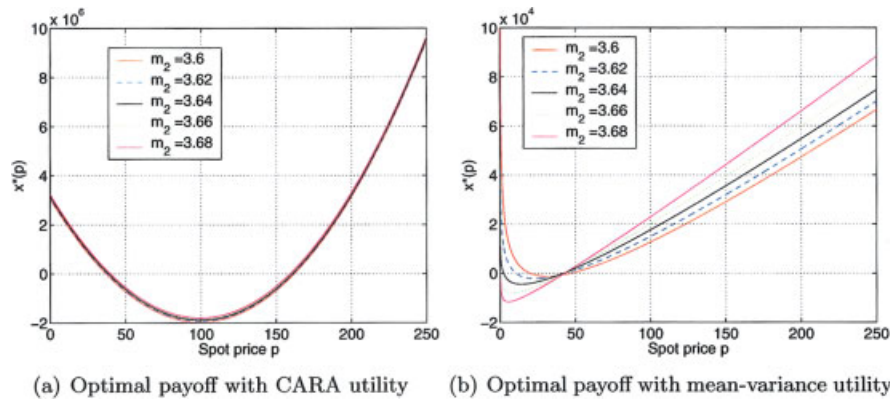
**Figure 4.** Optimal payoff functions for an LSE with mean-variance utility. (a) corresponds to  $(\log p, q) \sim N(3.64, 300, 0.35^2, 30^2, \rho)$ , and (b) corresponds to  $(\log p, \log q) \sim N(3.64, 5.77, 0.35^2, 0.09^2, \rho)$ . [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]



**Figure 5.** The graphs show numbers of forward and options contracts to be purchased in order to replicate the optimal payoff  $x^*(p)$  that is obtained for the LSE with mean-variance utility. In this example, the forward price is \$40.5/MWh; hence, the optimal portfolio includes the forward contract for  $x'(40.5)$  MWh, put options on  $x''(K)dK$  MWh for  $K < 40.5$  and call options on  $x''(K)dK$  MWh for  $K > 40.5$ . The upper Panels (a) and (b) correspond to price and load following a bivariate lognormal-normal distribution, and the lower panels correspond to price and load following a bivariate lognormal distribution. [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]



**Figure 6.** The comparison of profit distribution for an LSE with mean-variance utility for three cases: before hedge, after price hedge, and after price and quantity hedge, assuming the correlation coefficient between price and load to be 0.7. [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]



**Figure 7.** Sensitivity of the optimal payoff function to divergence between the risk neutral probability measure and the physical probability measure. The graphs correspond to the case when price and load follow a bivariate lognormal-normal distribution with correlation coefficient 0.5.  $m_2$  represents the mean of logarithm of price under the risk-neutral probability measure with  $m_2 = 3.64$  corresponding to the case  $P \equiv Q$ . [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]

affect the optimal payoff function above a certain level and the objective turns into minimizing variance. However, for smaller risk aversion, the mean-variance objective function puts more weights on the mean of profit; LSEs with low risk aversion will protect more against the lower spot price worrying that the expected profit is low from decreased load when spot price is low.

**4. CONCLUSION**

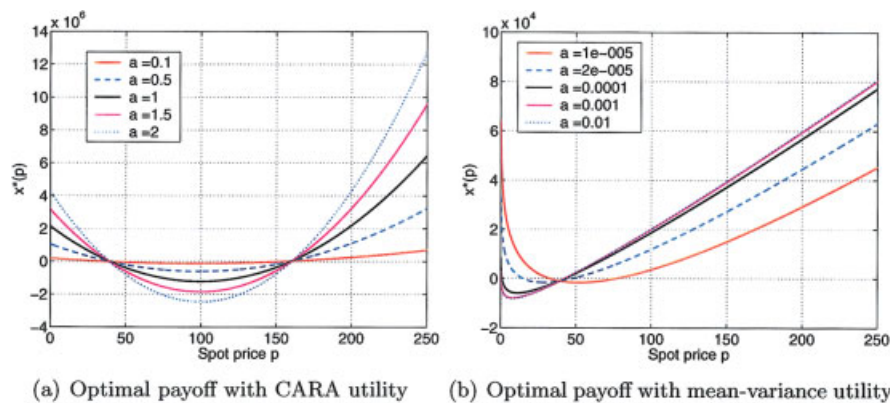
Price risk and its management in the electricity market have been studied by many researchers and is well understood. However, price risk should be understood as a correlated risk with volumetric risk (quantity risk), which is also significant. Volumetric risk has great impact on the profit of load-serving entities; therefore, there is a great need for methodology addressing volumetric risk management.

We discussed financial contracts that allow LSEs to mitigate volumetric risk: swing options, interruptible contracts,

and weather derivatives. In particular, weather derivatives are widely used to hedge volumetric risks since there are strong correlations between weather variables and power loads.

We propose an alternative approach that exploits the high correlation between spot prices and loads to construct a volumetric hedging strategy based on standard power contracts. In a one-period setting, we obtain the optimal zero-cost portfolio consisting of bonds, forwards, and options with a continuum of strike prices. Also, the paper shows how to replicate the optimal payoff using available European put and call options. The approximation of a payoff function using available options contracts that was shown in this paper can also be applied for hedging in markets that have put and call options with many different strike prices. The model and methodology are applicable to other commodity markets and with different profit functions.

There are more extensions that can be made to the current model. First, the zero-cost assumption allows the LSE



**Figure 8.** Optimal payoffs for the case when price and load follow a bivariate lognormal-normal distributions:  $N(3.64, 300, 0.35^2, 30^2, 0.5)$  under  $P$  while the log-price distribution is  $N(3.66, 0.35^2)$  under  $Q$ . [Color figure can be viewed in the online issue, which is available at [www.interscience.wiley.com](http://www.interscience.wiley.com).]

to borrow as much money at time 0 to buy the options contracts. Imposing credit limits on the hedging strategy would make the model more realistic. Second, the electricity market is incomplete, so the risk-neutral probability measure we choose would not be exactly the same as what the market uses for pricing. Therefore, a pricing error would exist, which can lead to inefficient hedging. A model that accounts for possible errors in choosing the risk-neutral probability measure would be a good extension for applications in the actual electricity markets.

## REFERENCES

- [1] N. Audet, P. Heiskanen, J. Keppo, and I. Vehvilainen, "Modeling electricity forward curve dynamics in the Nordic market," Modelling prices in competitive electricity markets, D.W. Bunn (Editor), Wiley Series in Financial Economics.
- [2] R.N. Boisvert, P.A. Cappers, and B. Neenan, The benefits of consumer participation in wholesale electricity markets, *Elect J* 15(3) (2002), 41–51.
- [3] G.W. Brown and K.B. Toft, How firms should hedge, *Rev Finan Stud* 15(4) (2002), 1283–1324.
- [4] P. Carr and D. Madan, Optimal positioning in derivative securities, *Quant Finan* 1 (2001), 19–37.
- [5] D. Caves, K. Eakin, and A. Faruqui, Mitigating price spikes in wholesale markets through market-based pricing in retail markets, *Elect J* 13(3) (2002), 13–23.
- [6] H. Chao and R. Wilson, Resource adequacy and market power mitigation via options contracts, UCEI POWER conference, Berkeley, March 2004.
- [7] CPUC Order Instituting Rulemaking to Establish Policies and Cost Recovery Mechanisms for Generation Procurement and Renewable Resource Development Interim Opinion, Decision No. 04-01-050, January 22, 2004.
- [8] CPUC Resource Adequacy Proceeding Interim Opinion Regarding Resource Adequacy, Decision No.04-10-035, October 28, 2004.
- [9] L. Clewlow and C. Strickland, Energy derivatives: Pricing and risk management, Lacima Publications, 2000.
- [10] D. Duffie, Dynamic asset pricing theory, Princeton University Press, Princeton, 1992.
- [11] A. Eydeland and K. Wolyniec, Energy and power risk management: New development in modeling, pricing, and hedging, Wiley, 2001.
- [12] T.W. Gedra and P.P. Varaiya, Markets and pricing for interruptible electricity power, *IEEE Trans Power Syst* 8(1) (1993), 122–128.
- [13] H. Geman, Towards a European market of electricity: Spot and derivatives trading, 2002, available at <http://www.nea.fr/html/ndd/investment/session2/geman.pdf>.
- [14] E. Hirst, Price-responsive demand in wholesale markets: Why is so little happening? *Elect J* 14(4) (2002), 25–37.
- [15] O. Jaillet, E. Ronn, and S. Tompaidis, Valuation of commodity based swing options, Working paper, UT Austin, 2001.
- [16] K. Kallsen, A utility maximization approach to hedging in incomplete markets, *Math Methods Oper Res* 50 (1999), 321–338.
- [17] I. Karatzas and S. Shreve, Methods of mathematical finance, Springer, New York, 1998.
- [18] J. Keppo, Pricing of electricity swing options, *J Deriv* 11 (2004), 26–43.
- [19] S. Koekebakker and F. Ollmar, Forward curve dynamics in the Nordic electricity market, *Manag Finance* 31 (2005), 73–94.
- [20] Y. Li and P. Flynn, A comparison of price patterns in deregulated power markets, UCEI POWER Conference, Berkeley, March 2004.
- [21] R.I. McKinnon, Futures markets, buffer stocks, and income stability for primary producers, *J Polit Econ* 75 (1967), 844–861.
- [22] G. Moschini and H. Lapan, The hedging role of options and futures under joint price, basis, and production risk, *Int Econ Rev* 36(4) (1995).
- [23] E. Nasakkala and J. Keppo, Electricity load pattern hedging with static forward strategies, *Manag Finan* 31(6) (2005), 115–136.
- [24] J. Rolfo, Optimal hedging under price and quantity uncertainty: The case of a cocoa producer, *J Poli Econ* 88 (1989), 100–116.
- [25] T.P. Strauss and S.S. Oren, Priority pricing of interruptible electricity power with an early notification option, *Energy J* 14(2) (1993), 175–195.
- [26] A.C. Thompson, Valuation of path-dependent contingent claims with multiple exercise decisions over time: The case of take-or-pay, *J Finan Quant Anal* 30(2) (1995), 271–293.
- [27] G. Unger, Hedging strategy and electricity contract engineering, a dissertation, the Swiss Federal Institute of Technology, Zurich, 2002.
- [28] M. Wagner, P. Skantze, and M. Ilic, "Hedging optimization algorithms for deregulated electricity markets," Proceedings of the 12th Conference on Intelligent Systems Application to Power Systems, 2003.