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FULL LENGTH PAPER

# A model and approach to the challenge posed by optimal power systems planning

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**Abstract** Currently, there is a need to plan and analyze the electric power transmission system in greater detail and over larger geographic areas. Existing models approach the problem from different perspectives. Each model addresses different aspects of and has different approximations to the optimal planning process. In order to scope out the huge challenge of optimal transmission planning, this paper presents a new modeling approach for inter-regional planning and investment in a competitive environment. This modeling approach incorporates the detailed generator, topology and operational aspects found in production cost planning models into a larger framework that can find optimal sets of transmission expansion projects. The framework proposed here can be used in an auction to award investment contracts or as a part of a more general policy analysis. The solution yields the set of transmission projects that have the highest expected benefits, while also representing generic generation expansions under the same objective. The model is a two-stage, mixed-integer, multi-period, N-1-reliable model with investment, unit commitment, and transmission switching. The combination of combinatorial, stochastic and operational elements means this model may be computationally intractable without judicious modelling aggregations

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or approximations to reduce its size and complexity. Nevertheless we show via a dual problem that analysing the economics and sensitivity of the solution is computationally more straightforward.

**Keywords** Duality · Integer programming · Stochastic programming · Generation unit commitment · Investment · Power system economics

**Mathematics Subject Classification** 90B15 Network models, stochastic · 90C11 Mixed integer programming · 90C90 Applications of mathematical programming · 91B32 Resource and cost allocation · 91B26 Market models (auctions, bargaining, bidding, selling, etc.)

## 1 Introduction

Today the investment in the US electric power system is about $800 billion with annual revenues of about $250 billion. Worldwide, these numbers increase by a factor of four or five. Because investment decisions related to the electric power system are large, even modest improvements in investment modeling can result in billions of dollars of cost savings. Such potential indicates the need for improvements to modeling the electric power planning and investment processes.

Historically, planning has evolved from a process in which investment decisions were made centrally by a vertically integrated utility in consultation with its neighbors, to a process in which investment decisions are more decentralized and potentially more competitive. High voltage transmission proposals often impact a large geographic area spanning more than a single utility or state. Existing approaches to transmission planning and investment have implicit and explicit assumptions and approximations that need to be re-examined in the context of a smarter grid and increased amounts of energy from wind and solar generators, batteries, and demand-side market participants. Some approximations and assumptions in current models were necessary to make the problem computationally practical for the technology that was available when computer-assisted planning started decades ago. Other assumptions and approximations were made to simplify uncertainty, including failure modes and demand growth. Still other assumptions and approximations were made in order to harmonize planning and investment approaches with the market design de jour. Many of these assumptions and approximations limit advancements in optimal inter-regional planning of the grid.

Reliability is a process of creating rules and penalties for non-compliance to reduce the probability of cascading blackouts (blackouts caused by disturbances in other areas) and serious equipment damage. Cascading blackouts affect large geographic areas and their prevention is a public good for that area. Historically, reliability standards were guidelines and compliance was voluntary. Steps to formalize, standardize and computerize reliability started after the 1965 Northeast Blackout. Generally, reliability was confined to a vertically integrated utility and was a weakly defined concept that often included considerable judgment. Many planning models were developed as reliability models and still reflect a reliability approach. As a result of 2003 Northeast Blackout and the subsequent legislation (EPACT 2005), the Federal Energy Regulatory Commission now has the formal authority to regulate and enforce reliability standards. For N-1 reliability, the system must be stable and able to survive the failure of any one asset with a high probability. Reliability includes numerous other rules including situational awareness, vegetation management, and operator training that are not considered here. Production cost models, which simulate unit commitment and economic dispatch operations, are often used in economic studies of proposed transmission expansion projects.

With the advent of large amounts of wind and solar, along with storage and more price-responsive demand, the current approaches need to be modified. Today, for computational and management reasons, models are decomposed, compartmentalized and reduced in size using a mixture of engineering judgment, experience and off-line modeling. Planning results are tested for adequate voltage stability, inertia and various other aspects of reliability. Over time as the data, hardware and software for solving the problem improve, more constraints can be modeled explicitly over larger regions. With experience, the solution times can be reduced and better modeling can be introduced.

The approach presented here integrates aspects from production cost models and investment models. Our primary objective is twofold. First, we use a model to scope out the challenge that is faced in optimizing transmission expansions over alternatives specified by the analyst or planner. Second, we use the model to present the enormous complexity of the problem to the optimization community because electricity stakeholders need a new generation of much more ambitious and higher performing numerical software to sensibly discuss optimal expansion in transmission capacity.

Our approach chooses the transmission investments that give the highest expected net benefits to society while recognizing N-1 reliability constraints and environmental goals. The model also recognizes generic generation investment alternatives, and co-optimizes these expansion costs with transmission expansions. If demand is inelastic, the overall objective is to achieve the expected lowest cost of transmission and generation investments that achieve specified reliability levels and environmental goals. The analyst or planner can modify constraints to represent different policy scenarios. For example, different renewable portfolio standards can be specified and the resulting optimal transmission expansion (from among prespecified alternative projects) is produced as an output. The expected long term effect on the mix of generation can also be analyzed via the inclusion of generic generator expansion options. To the extent possible, the generation and transmission options presented to the model should consider regulatory difficulties by either excluding certain technologies, or by reflecting the difficulties in the costs of construction. Multiple objectives can be included via constraints, additional costs or explicit tradeoffs. For example, environmental objectives can limit the amount of emissions, such as $CO_2$, $SO_2$, or Hg, using model constraints. The model satisfies the modeling criteria of many environmental groups.[1] The model is policy flexible in that the geographic scope, demand responsiveness and energy efficiency capabilities can be defined exogenously, but optimized endogenously.

[1] Proposed Framework for Electricity Grid Planning, Discussion Draft, October 9, 2009, issued by representatives from the National Audubon Society, Conservation Law Foundation, Energy Future Coalition, ENE-Environment Northeast, Environmental Defense Fund, Natural Resources Defense Council, Piedmont Environmental Council, Sierra Club, Sustainable FERC Project and Union of Concerned Scientists.

The model allows as input potential transmission investment. Generic generation construction, installation and operation costs are estimated for different facility types, sizes and locations. Generic generators are chosen by the model to ensure optimality and analyze the effect of future transmission investment. The approach is used to better inform the decision making process, and select optimal transmission investments from among the possible combinations. Over time the market is assumed to fill in the generation needs optimally.

The modeling approach presented is an extension of the model for a day-ahead market with transmission switching (see Hedman et al. [1–3] and [4]; also O'Neill et al. [5,6] and Fisher et al. [7]). New investments are modeled by extending the unit commitment formulation to include investment decisions, and by incorporating transmission switching decision variables to include investment decisions. The main differences are the inclusion of investment decision options, additional uncertainties and a longer time horizon. The process also considers changes to existing resources and infrastructure, to the extent that such changes increase system efficiency. The solution of a large scale version of the problem would benefit from advancements in hardware and optimization software. The model can be made more (or less) granular with an increase (or decrease) in the computational burden. A consolation is that solution time window is measured in days not hours and computer hardware and software continue to improve at a rapid pace.

In Sect. 2, we present an overview of models and approaches for transmission planning analysis. In Sect. 3, we present an overview of the proposed approach and model. Input data and sensitivity analysis are discussed, as well issues surrounding transmission investment and transmission costs and rights allocation. In Sect. 4, we present the stochastic multi-period N-1-reliable forward market model with unit commitment, transmission switching and investment. The mathematical formulation is presented, along with economic analysis of the dual problem. Finally, we conclude and summarize in Sect. 5.

## 2 Literature and model review

In academic literature, a number of optimization based approaches have been presented for transmission expansion planning. Many of these approaches do not guarantee optimal solutions but instead use approximations necessary to handle the magnitude of the problem, and the computational difficulty presented by binary investment decisions. Garver [8] and Villasana et al. [9] presented linear programming approaches for finding feasible transmission network expansions given future loads and generation. Dusonchet and El-Abiad [10] discussed the use of dynamic programming to deal with the size and complexity of a transmission planning optimization problem.

Romero and Monticelli [11] proposed a method for solving network expansion planning problems with linear and mixed integer programming techniques, by relaxing the problem to a transportation model without integrality constraints and then successively introducing the complicating constraints to move towards a final solution. Baughman et al. [12] discussed models for the inclusion of transmission expansion decisions in integrated resource planning (IRP). Gallego et al. [13] presented a genetic algorithm approach for solving the transmission system expansion problem. Binato et al. [14] explored a Benders decomposition approach to solving mixed integer programming problems for the transmission expansion problem. Alguacil et al. [15] proposed a mixed integer programming formulation of the long term transmission expansion problem with binary transmission investment decisions were represented by variables and applied it to a 46 node single period model of the Brazilian power system. DeOliveira et al. [16] presented a sigmoid function approach for binary investment variables in the optimal transmission expansion problem, and tested it on a model of the southeastern Brazilian system. De la Torre et al. [17] presented a mixed integer program for long term transmission investment planning in a competitive pool based electricity market.

Moulin et al. [18] discuss an integer programming formulation for transmission expansion planning which allows for re-design of the existing network in addition to investment decisions for new assets, since the properties of electric transmission networks can allow for lower cost solutions when some combination of transmission elements are removed when others are added. Kazerooni and Mutale [19] solve the N-1 security constrained transmission expansion optimization problem while incorporating environmental constraints. van der Weijde and Hobbs [20] formulate a two-stage stochastic program in which generation investment is a stage two variable, and is allowed to react to the actions of transmission planners. They use the model to find minimal investment costs, as well as to evaluate the value of information and flexibility in planning for renewables through simulations involving the power system in Great Britain.

For several reasons, the approach presented here does not contain an explicit game theoretic aspect such as those found in Sauma and Oren [21,22] and Murphy and Smeers [23,24]. First is that game theoretic analysis adds an additional computational burden to an already difficult problem. Second, almost all show inefficiencies that, in theory, call for intervention if intervention is cost justified. In the US, commodity markets are mitigated and transmission markets are predominately cost-of-service with free rider problems. The approaches to date have been too abstract. Third, game theoretic approaches are often very complex and require many assumptions that move them away from the actual markets. Here, we are trading off the incentive problems of IRP with market mechanisms such as auctions with better incentives. Also, we present a simplified cost allocation for the optimal transmission expansion plan.

Currently, there is also a range of commercial modeling tools that fall under the category of planning models. These include production cost planning models that simulate unit commitment and economic dispatch, which are widely used in Regional Transmission Organization (RTO) and Independent System Operator (ISO) planning and are available from a number of vendors. Production cost modeling is one of the predominant methods for evaluating the economic benefits of transmission expansions on the power system. Examples of commercial production cost models are Gridview [25], Multi Area Production Simulation (MAPS) [26] and PROMOD [27].

RTOs and ISOs use commercial productgion cost models, along with internal software, for planning analysis. The current framework for production cost modeling involves simulations of the unit commitment and economic dispatch process for a chosen footprint (single area to interconnect) and time frame (weeks to years). The modern commercial grade production cost software for economic evaluation of transmission investments is often limited to a linear approximation of the alternating current optimal power flow (ACOPF) problem, which represents the network flow model for the electric transmission grid, called the direct current optimal power flow (DCOPF) problem. The ACOPF is a non-linear, non-convex problem involving trigonometric functions whereas the DCOPF is a linear program if the costs are represented through piecewise linear functions. While it is acceptable to make such an approximation, in regards to the ACOPF, for long-term planning models based on todays technology, such software have many other coarse approximations as well. For instance, such packages are often merely able to solve linear programs. Modeling requirements that require binary variables may be handled by heuristics and gross approximations.

Binary decision modeling should involve a mixed integer programming framework, which is often lacking in key areas. As an example, a set of new transmission and generation projects can be analyzed as specified (exogenously) by the user, but the software would not be able to identify the optimal set of projects, and the optimal timing of the investment, from a specified set of alternatives. Such abilities may be useful, for example, in the context of analysis to support interconnect wide planning for the integration of renewable resources. This is not just the case for transmission planning but it is also the case for transmission maintenance scheduling. Maintenance scheduling for transmission and generation is important to take into consideration within long-term investment planning models. Even though these problems require a mixed integer programming framework, gross approximations are made by implementing rule-of-thumb policies. ISONE released a report stating that they are saving tens of millions of dollars a year by considering the economic impacts of transmission outage planning instead of only considering the reliability aspects [28]. However, the methods that they are using rely on taking into consideration the prices in the market to estimate the economic impacts of transmission maintenance schedule, which then asks the question regarding how much more could be saved if a more direct approach is taken in the future.

Another limitation of modern commercial grade packages is their ability to properly model energy storage capabilities and their participation in buying and selling energy. Techniques used by these packages to determine pumped-storage facility production and consumption (such as peak-shaving techniques) can produce anomalous and inaccurate results, for instance facilities consuming at high prices and producing at low prices.

Overall, todays commercial grade technology for this complex combinatorial problem is not even at the level of our capabilities to date. With the advent of intermittent renewable resources and new smart grid technologies, the need for new modeling and algorithmic approaches for this problem is even more pressing.

## 3 Overview of proposed approach and model

Optimal transmission planning modeling decides the expected best set of transmission investments based on possible future scenarios and the probabilities of their realization. The model and process in a nutshell is:

1. Develop, standardize and refine data needs for planning
2. Decide on a set of possible future scenarios and their associated probabilities.
3. Take transmission proposals
4. Solve a stochastic mixed integer program to find the investments with the highest expected net benefits
5. Perform sensitivity analysis on scenarios, analyze the optimal set of transmission investments in each scenario
6. Determine the beneficiaries
7. Allocate transmission costs and rights
8. If satisfied with the results, stop; otherwise, go to 1.

Unless each step is accomplished at a certain level of quality, the overall process will likely fail. In the following sections, we discuss the steps in more detail.

### 3.1 Input data, assumptions and scenarios

Good data is necessary to achieve good modeling results. It goes almost without saying that a high fidelity database is a necessary prerequisite for good results. For large areas crossing state lines and company boundaries, coordination and standards development are important.

The paradox or irony of risk managed forecasting or planning under uncertainty is that the ex ante optimal solution is almost always wrong ex post. The simple reason is that risk managed approaches consider and respond to scenarios that ex-post did not happen and did not need to be considered.

Nevertheless, there is no good substitute for good scenario planning. Good scenarios are the result of vigorous transparent public debate. Scenarios need to focus on assumptions about technological innovation, environmental issues, input prices and the probability of each scenario. Technological innovation and scientific discoveries have perplexed forecasters for centuries. The assumptions about technological innovation can radically change the model outcomes.

Controversial but important scenario parameters are the future prices of carbon emissions (or amount of carbon emissions permitted), coal, oil, and natural gas. The Energy Information Administration (EIA) produces annual long term forecasts. They are generally considered the default assumptions in analysis. This is not because EIA necessarily gets it right, but because they are considered the least biased and have the best information base. Some policy objectives are exogenously determined and can be incorporated with constraints. Each environment pollutant (for example, $CO_2$, $SO_2$, or $NO_x$) can be constrained in any geographic region. Each resource (for example, wind, solar or geothermal) can be required for any geographic region.

Scenarios and their probabilities must be developed using forecasts and input from industry experts. Developing the probability of some scenarios can be a difficult task. For example, it is very hard to estimate the likelihood that renewable portfolio standards or environmental policies will impose new requirements at a later date, or the possibility of a future spike in fuel prices. All scenarios involve forecasting that is inherently incorrect. Sensitivity analysis that examines outputs under a range of different scenarios will always be a critical part of the process.

The smarter grid is more controllable and flexible, and makes more efficient use of existing and new assets. Today, only limited control and flexibility of electric assets are built into electric network optimization models. In our model, we include smart grid assumptions as granularity allows. Topology optimization and corrective switching are examples. Assumptions about the smart grid can change the outcomes and even the modeling approach. Also, the significant penetration of wind and solar introduces new stochastic variation that could cause both operational and market design problems without the smart grid. For example, wind power and electric cars present interesting peak/off-peak issues.

### 3.2 The stochastic mixed integer programming planning model

The planning model in this framework is a stochastic two-stage mixed integer program. The objective of the model is to maximize the expected market surplus (benefits to society) from new and existing investment. The investment decisions are binary variables. A new generator or transmission asset cannot be in the network unless the investment has been made to construct. For each state or contingency, the operating variables and associated constraints assure the feasibility given the investment decision. Transmission switching is relevant because if a valuable line could be blocked by a low capacity line in a circuit then the low capacity line can be removed to improve the market performance.

Each generator has operating costs, startup, minimum up time, shutdown and minimum downtime constraints and operates with upper and lower bounds within ramp rate constraints. Transmission assets can be switched in and out of the network and can operate continuously within upper and lower bounds. Reliability is met with N-1 DC reliability constraints for both transmission and generation. Transmission switching is assumed to be available in contingency scenarios. This is a realistic assumption in cases where Special Protection Schemes (SPS) are in place for certain scenarios. The probability of two or more simultaneous contingencies is not explicitly modeled; that is, the market will not clear with specified probability. The probability of two or more simultaneous outages are calculated by treating outages of system elements as independent events.

Load is modeled comparably to generators. If the load chooses to be explicitly price-responsive (bid into the market), it has comparable bidding parameters to generation. For example, load can bid the value of consumption in a single period or can bid a single value for an entire eight hour shift using minimum run parameters. Price-responsive demand is not included in reliability calculations since demand will voluntarily curtail itself when the price is too high.

System constraints include Kirchhoffs first law, power balance equations at each bus, Kirchhoffs second law, flows around a circuit, and phase angle differences. In a state when the transmission element is not in the network the constraints are adjusted. If the transmission element is in the network then the second law is enforced. Both AC and DC lines are modeled. For renewable portfolio requirements, we add the necessary parameters, variables and constraints for either production or capacity requirements.

Due to the length of the horizon, cost and values are discounted. This allows the model to optimize the expected discounted present value of the investments. Fortunately, discounting makes errors in later years, where input assumptions are more vulnerable to error and uncertainty, have lesser impact than assumptions in earlier years. We also assume risk neutral market participants because the cost of transmission investment is paid for, in part, under a cost-of-service regime that reduces the investment risk.

### 3.3 Sensitivity analysis

Different approximations can be used for different purposes. For example, smaller weaker-fidelity models can be used to solve many rough cut scenarios quickly in preliminary or high-level sensitivity analysis. Larger, higher-fidelity models can be used to ensure the detailed or final decisions on investments, contracts and cost-allocation approach are consistent with smaller models. The results can be fed back and forth to refine the models.

Sensitivity analysis can address many issues including sensitivity to data inputs, assumptions, and approximations. The list of possible sensitivities is large and can be computationally intense. For the purpose of economic analysis, sensitivity analysis is conducted on two levels. Marginal economic information is a model output. The first is a marginal analysis of continuous variables and parameters. For example, the marginal cost of a renewable portfolio constraint is a model output, that is, the level of subsidy needed to achieve an equivalent result. The second is the incremental analysis of binary decision variables. A set of binary variables can be changed, the model is then resolved, and the change in the objective function then gives the expected incremental value for the change in that set of binary variables. For complete sensitivity analysis, many combinations of binary variables may need to be considered, but this is practically impossible for all but small models. For practical reasons we need to find analysis that is cost beneficial and present an approach later in this section.

We recommend three to five working scenarios: for three scenarios a most likely scenario (70–80 %) and a bounding scenario (10–15 %) on each side. The five working scenarios consist of a most likely scenario and two bounding scenarios on each side. In general, the entire process is iterative. That is, the model is fined tuned as experience with the model grows.

We suggest starting with a weak approximation of five year increments, four seasons, and sample days (peak, off-peak and two shoulder periods). For fossil generators, we suggest steady-state operating conditions—startup only for peaking generators, minimum and maximum operating levels, and average variable operating costs. For variable generators, we suggest variable costs and 3-part partitions of event space for generator output. For transmission, we suggest maximum steady-state capacity.

### 3.4 Transmission investment decisions

In this model, investments are not made unless they are a part of the optimal solution. The model can be used in several modes varying from advisory/insight status to an auction mode. In the advisory mode the model is used to guide the planning process. In the auction mode it is used to decide what to build and what to pay.

In the auction approach, the generic contract details and bidder qualifications are specified prior to the auction. Market participants submit offers to build with costs and technical specifications of the proposed investment. The model finds the expected optimal (or near optimal) transmission plan. Conceptually, this is similar to the energy capacity market, but more complex. The winning transmission projects are awarded contracts to build the assets at their offer costs. We recommend that the auction be a first-price or pay-as-bid auction with ex-ante market power mitigation because a second price auction presents issues that are computationally intense and may be unnecessary if entry is competitive. Also, with market power mitigation, the value of the second price auction diminishes. The winning generic generation is not awarded a contract. Competition should determine who builds the generation although the new transmission will in part determine where new generation is located and what is built.

In either mode, transmission projects compete with each other and drive down offer costs. Transmission projects can compete with other projects to build the same asset. Competition also comes from combinations of other proposals including local generation. Sensitivity analysis can help determine if there is market power. We now proceed to address the allocation of transmission investment costs and transmission rights.

### 3.5 Transmission costs and rights allocation

Cost allocation is a part of setting just and reasonable rates as the law requires. The reality of planning causes the benefits to be uncertain due to uncertainty surrounding the data, model approximations and scenario specifications. The uncertainty in the data and model approximations can usually be improved from research, but scenario specification has inherent uncertainties.

An important question is not whether it is possible to do an analysis of benefits and beneficiaries, but whether the process can be improved upon. Additional sensitivity analysis may be helpful in resolving allocations. There are general approaches for cost allocation: beneficiaries pay, winners compensate the losers, postage stamp, highway/byway, voltage level and the Argentina method [29].

Some argue that transmission expansion is a public good. Since each transmission asset has a finite capacity and can become congested, it should not be characterized as public good.[2] Because the transmission assets can cause significant network externalities, they also should not be characterized as private goods. They should instead be characterized as club goods.[3] We propose multi-part pricing including capacity and usage. One part is a capacity right or option call on capacity with a low or zero strike

[2] A public good has the properties that it is not possible to prevent others from consuming the good and the consumption by one does not prevent consumption by others. A private good has the properties that it is possible to prevent others from consuming the good and the consumption by one prevents consumption by others.

[3] A club good has the properties that it is possible to prevent others from consuming the good and the consumption by one does not prevent consumption by others.

price. The second part is a usage charge or opportunity cost.[4] Transmission rights are tradable. Transmission rights allocations are difficult, even in a simple model. Here we auction the created transmission rights, that is, flowgate rights. Flowgate rights can also be allocated to help compensate the losers. Flowgate rights can be implemented in both the Financial Transmission Rights (FTR) and point-to-point environments. FTRs and point-to-point rights can be defined as portfolios of flowgate rights. The principal difference between point-to-point tariffs and FTR tariffs is that the point-to-point tariffs allow the capacity to be withheld at offers less than the maximum rate. This is not allowed with FTR tariffs. Point-to-point rights are similar to FTR options.

We could also use an auction for allocating and reallocating transmission rights. In the first round, the bid currency could be the cost allocated to the market participant. This approach to transmission rights allocation could be similar to the process for allocating transmission rights currently in RTO and ISO markets.

When cost allocation disagreements occur, usually the strongest disagreements are in allocating costs to market participants not expected to benefit or not allocating cost to those who benefit (free riders). Since the true information on benefits to any market participant may be unknown even to the market participant, and because costs are allocated in proportion to benefits, market participants if asked may attempt to understate the benefits. Here we simply assess benefits as the change in costs of energy at a specific bus or node in the model with and without the new investments.

Conceptually, there is a general agreement and a circuit court decision[5] that beneficiaries of transmission should pay for the transmission and receive the associated transmission rights. There are significant disagreements on what this means, how much each market participant should pay, how the rights are allocated and the resulting externalities. Cost allocation occurs after uncertainties and disagreements on scenarios, assumptions and approximations have arisen in getting to the optimal transmission plan. This uncertainty combined with siting issues presents significant potential political problems for both choosing projects and cost allocation. When the market participants are not able to agree on allocation rules, a higher authority must impose them.

Cost allocation is a part of cooperative game theory that allows the participants to form into groups to cooperate and negotiate the benefits or costs [30].[6] Cost allocation can be highly mathematical (for example, using the Shapley value, Nucleolus, core, kernel, etc) or highly intuitive or behavioral (for example, consider traditional approaches, focal points and notions of fairness). Here we will focus on the simpler more transparent approaches and reject the more mathematically complex and computationally intensive approaches.

[4] Firm and interruptible service can be created under these tariffs, similar to point-to-point service under the Order 888/890 tariffs with a fix of the contract path approach using flowgate rights portfolios.

[5] For example, see Illinois Commerce Commission, v. Federal Energy Regulatory Commission, United States Court of Appeals for the Seventh Circuit, August 6, 2009.

[6] Cooperative game theory contrasts with noncooperative game theory where market participants are not allowed to communicate explicitly with each other. Most competitive markets are analyzed under the noncooperative game theory paradigm, for example, a Nash or perfect equilibrium is a common model for deciding the optimal expansion.

An important caution in cost allocation in a multi-project environment is that the value is for all projects to be taken as a whole. The value for all projects to be taken as a whole is not the sum of the individual value of each individual project. The benefits of an individual project can be examined by solving the model without a specific asset. At the individual project level, there is a greater chance that a project must be considered in concert with other project or projects in order to achieve its optimal value.

We present a simple model of cost allocation. First, we calculate the difference in the expected costs of energy at each bus with and without the new investments. This is a relatively easy problem to solve since the investment decisions are fixed from a planning optimum.

Let *DTR* be the new transmission rights created by the expansion and *TEC* be the total expected cost of the optimal transmission investment. Auction the *DTR*, receiving *RTR*. If *RTR*–*TEC* $>= 0$, no cost allocation is necessary.

Let *SB* be the incremental efficiency gains or system benefits from the set of transmission projects. By the result of the optimization, since one option is not to build, $SB > TEC > 0$; otherwise, the set of projects would not be selected for investment. Let $B_i$ be the difference between the expected costs of energy under no investment for market participant or defined group of market participants $i$. Groups would normally be defined regionally. $B_i > 0$ corresponds to lower costs of energy for a grouop of buyers $i$ or to higher costs of energy for a group of sellers $i$ under the optimal investment as compared to no investment. We present two allocations schemes: one where the winners compensate the losers and one where winners do not compensate the losers.

*A scheme where the winners compensate the losers.* Let $\{1, \ldots, I\}$ partition the market participants into groups.[7] Let $TB = \sum_i B_i$. We define $s_i = B_i/B$, $\sum_i s_i = 1$. Let $NC_i = s_{iTEC}$. $NC_i$ for $i \in \{1, \ldots, I\}$ is a cost allocation scheme. In this scheme, the costs are allocated in proportion to the net benefits from the investments. Winners compensate the losers, that is, if $s_i < 0$ (which implies $B_i < 0$), market participant $i$ receives a payment from the allocation process or perhaps a reallocation of existing costs.[8]

*A scheme where winners do not compensate the losers.* Let $BW = \sum_{B_i>0} B_i$, $s_i = max(B_i/B, 0)$, $\sum_i s_i = 1$ and $TEC_i = s_{iTEC}$. $TEC_i$ for $i \in \{1, \ldots, I\}$ is a cost allocation scheme where winners do not compensate the losers (that is, losers receive no compensation, and winners are allocated a proportionate amount of costs).

In Fig. 1, we present a simple example similar to one presented by Hogan [31]. All costs and benefits are expected values. To keep a simple example, we assume that all market participants buy and sell at the nodal price and transmission rights (*TR*) are paid at the flowgate marginal price. In general we allow hedge contracts, but not in this simple example since it renders the example more complicated. The pre-expansion transmission capacity of $q^{max}$ provides benefits to the import region represented by area $A$. The existing transmission benefits are area $D + B + G$ and the benefits to the export region are area $J$. $q'^{max}$ is the post-expansion capacity with total transmission

[7] A defined group of market participants could be a former vertically integrated utility, an entire state or an individual market participant. If this grouping does not cross state boundaries, states could allocate costs within the group.

[8] For example, see SPP Balanced Portfolio approach. http://www.spp.org/section.asp?pageID=120.

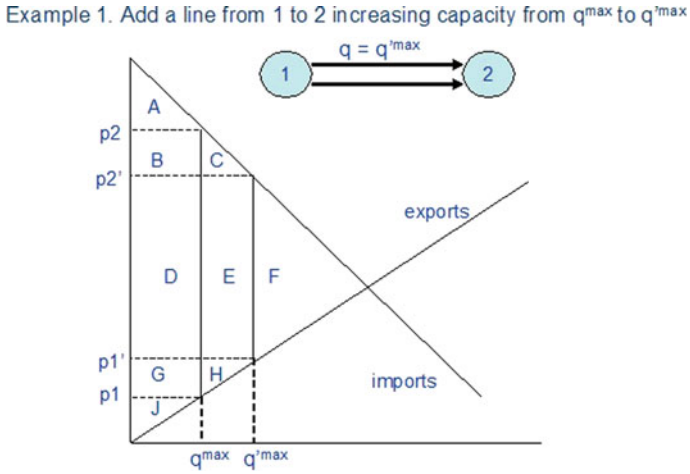


**Fig. 1** Impact of adding a line to increase capacity

expansion costs TEC. The incremental benefits to the import region are area $B+C$ due to a reduction in price. The expected incremental transmission right benefits are area $E$ if the result of the expansion is little or no congestion, but since this is an optimal expansion the additional incremental costs of decongesting the system are greater than $F$.

The efficiency criterion to build is when the change in market surplus and net system benefits $C + E + H > TEC$. The incremental benefits to the export region are area $G+H$ due to an increase in price. $B$ and $G$ are pecuniary post-expansion benefits, that is, they are transfers from existing *TR* holders not efficiency gains from the expansion.

If $E \gg TEC$, the auction of transmission rights cover costs or merchant transmission will build for the expected incremental *TR* revenues. The existing flowgate rights holders lose value $B + G$ or on a unit basis the value shrinks from $p_2 - p_1$ to $p_2' - p_1'$. If the flowgate rights holders are hedging spot transactions against charges for transmission congestion associated with a generation contract from 1 to 2, the losses in flowgate rights revenues will offset the nodal price change. If we auction transmission rights and auction revenues as *RTR*, we need to collect *TEC–RTR* from regions 1 and 2.

In non-price regulated markets, the winners do not generally compensate the losers. If the change in market surplus due to the expansion is $C + E + H > TEC > E$, merchant transmission will not build, but the expected benefits (efficiency gains) exceed the expected costs. The import region benefits are $B+C$, the transmission rights benefits are $E$, and the export region benefits are $G+H$. Since $B+C+E+H+G \gg TEC$ and there are expected benefits to all participants, there are numerous possible cost allocations. If the market participants have accepted the model assumptions and are risk averse, an acceptable cost allocation is possible without regulatory intervention; however, this will not keep market participants from arguing for smaller allocations of cost. It is well known that identifying the beneficiaries and agreeing upon the resulting benefits are considered to be major challenges with respect to transmission investments today.

In price regulated markets, winners can be forced to compensate the losers. The losers can include the original transmission rights holders. Currently transmission rights in ISOs are for 10 years or less. Hedgers may be indifferent to the lost value in transmission rights if they are gained back in price reductions for buyers and price increases for sellers.

## 4 The mathematical model

In this section we present the detailed mathematical model description. The planning model is a stochastic two-stage mixed-integer program. The objective of the model is to maximize the expected market surplus (benefits to society) from investment. The stage 1 binary variables are investment decisions and other decisions that must be made before the uncertainty is resolved. For each state, the stage 2 variables and associated constraints assure the market feasibility (reliability) of the investment decisions and the stages contribution to the expected market surplus. The market solution is reliable if the solution feasible. The market solution is economic if the solution is optimal. Here, we will use the terms, bid and cost or value, interchangeably. Value is the negative of cost.

A DC representation of the transmission system is used. Transmission assets can be switched in and out of the network inside the time step of the model and can operate continuously within upper and lower bounds. Transmission switching is relevant because a new line could be blocked by a low capacity line in a circuit formed by the new line, where removing the line improves the market performance.

This formulation quickly can balloon in size and computational complexity making it important to reduce its size without over-comprising fidelity. The model can be simplified in various ways. Simplifications include changing the granularity in topology, time step and probability event space partitions. Also, some binary variables can be converted to continuous variables. By not endogenously modeling subsets of the event space, for example, two or more simultaneous outages, the market will not clear with a specified probability.

Each state is a realization from a convolution of probability distributions including demand and variable energy resources, generation and transmission outages, and exogenous input parameters, for example, fuel prices. Some events can be excluded from the model and approached via sensitivity analysis. For example, the probabilities for completion of facilities such as transmission facilities, coal and nuclear plants that have a more difficult time clearing environmental hurdles, are not endogenously modeled, but can be approached in sensitivity analysis. Also, specific fuel prices and environmental scenarios can be modeled as exogenous scenarios. Other analyses such as voltage and transient stability are performed subsequent to the investment model decisions. This process embeds the model in a larger process. Next, we focus on the formulation of the model.

### 4.1 The model

We start by defining the sets, variables, and parameters used in the model. Next, we formulate the model. The model shares the formulation of transmission switching and unit commitment problem presented (see for example, Hedman et al. [1,2] and O'Neill et al. [6]). In addition we add a stochastic parameter, $\rho_c$ (state probability) to the objective function, forming a two-stage stochastic program for N-1 reliability. Another principal modification of the day-ahead market model is how the scenarios are chosen and the probabilities assigned to each scenario. We include the costs of adjusting generation in a transmission outage, but since a transmission outage is 10–100 times less likely than a generator outage it may have little affect the optimal solution and could be omitted. Prices for fuel and environmental pollutants become input to cost functions.

Different reserve services for both transmission and generation have been presented, which are based on the deterministic market clearing. Traditionally, security-constrained models include the line flow constraints with pre-specified areas for reserves. We present a model where these constraints can be explicit and decisions are made endogenously.

## 4.2 Nomenclature

### Indices

$AC$: AC transmission elements
$DC$: DC transmission elements
$k$: transmission element (line or transformer), $DC$ or $AC$, $k = AC \cup DC$
$n, m$: nodes; $k(n, .)$ is the set of elements with $n$ as the 'from' node; $k(., m)$ is the set of elements with $m$ as the 'to' node; $g(n)$ is the set of generators or load at node $n$.
$t$: time period; $t \in \{1, \ldots, T\}$.
$c$: contingency or state index for scenarios

### Primal Variables

$\theta^+_{kct}, \theta^-_{kct}$: non-negative voltage angle difference in the $n$ to $m$ or $m$ to $n$ direction.
$\theta_{mct}, \theta_{nct}$: voltage phase angle at from bus $m$ and to bus $n$ for transmission element
$\theta_{kct}$: voltage angle difference. $\theta_{kct} = \theta^+_{kct} - \theta^-_{kct} = \theta_{mct} - \theta_{nct}$ from $n$ to $m$.
$P^+_{kct}, P^-_{kct}$: non-negative real power flow from node $n$ to $m$ or node $m$ to $n$ on element $k$.
$P_{kct}$: real power flow from node $n$ to $m$ on element $k$ $P_{kct} = P^+_{kct} - P^-_{kct}$
$P_{gct}$: real power supply from generator (>0) or demand from load (<0) $g$ at node $n$.
$r^+_{gct}, r^-_{gct}$: ramp rate in the up or down direction for generator (or load).
$x_{gt}$: binary investment decision for generator (or load) (1 for investment, 0 otherwise).
$u_{gt}$: binary unit commitment for generator (or load) (0 down, 1 operational).
$v_{gt}$: startup decision for generator (or load) (1 for startup, 0 otherwise).
$w_{gt}$: shutdown decision for generator (or load) (1 for shutdown, 0 otherwise).
$z_{kct}$: binary switching decision for transmission element $k$'s inclusion in the topology (1 for in, 0 out).

$sw_{kct}$: variable to capture the change in the status of a transmission element (open-closed or closed-open) from the previous period.
$y_{kt}$: investment decision for transmission element (1 for investment in $t$, 0 otherwise).

**Dual Variables**

$\alpha^{+}_{nct}, \alpha^{-}_{nct}$: the marginal value of raising or lowering the maximum nodal phase angle.
$\lambda_{nct}$: the locational marginal value of generation or load.
$\eta^{+}_{kct}, \eta^{-}_{kct}$: the marginal value of increasing the flow limit (positive direction) or reducing the flow limit (negative direction) on a transmission element.
$\mu^{+}_{kct}, \mu^{-}_{kct}$: the marginal value of the susceptance of AC transmission element with positive or negative direction flow.
$\kappa_{kt}$: the marginal value of enforcing the relationship between investment and switching decisions for a transmission element.
$\beta^{+}_{gct}, \beta^{-}_{gct}$: the marginal value of an additional unit of the maximum level or reducing the minimum level of generator (or load).
$\omega^{+}_{gct}, \omega^{-}_{gct}$: the marginal value of ramping up or down for generator (or load).
$\chi^{+}_{gct}, \chi^{-}_{gct}$: the marginal value of another unit of ramping capacity up and down for generator (or load).
$\tau_{gt}$: the marginal value of enforcing the relationship between startup, shutdown, and unit commitment variables for generator (or load).
$\xi_{gt}$: the marginal value of enforcing the relationship between investment and startup decisions for generator (or load).
$\psi_{t}$: the marginal cost of the renewable production portfolio standard.
$\tau_{t}$: the marginal cost of the renewable portfolio capacity standard.
$\delta_{kt}$: the marginal value of switching a transmission element.
$\pi_{kt}$: the marginal value of enforcing the investment decision for a transmission element.
$\sigma_{gt}$: the marginal value of enforcing the startup value for generator (or load).
$\gamma_{gt}$: the marginal value of enforcing the shutdown value for generator (or load).
$\varphi_{gt}$: the marginal value of enforcing the investment decision for generator (or load).

**Parameters** (Note: all monetary values are assumed to be properly discounted, and every cost must be indexed by t.)

$I_{gt}$: total investment cost for generator (or load); generally $I_{gt} \geq 0$
$I_{kt}$: total investment cost for transmission element; generally $I_{kt} \geq 0$
$\theta^{+}, \theta^{-}$: maximum and minimum voltage angle difference; for convenience we assume $\theta^{+} = -\theta^{-}$.
$P^{max}_{gct}, P^{min}_{gct}$: maximum and minimum capacity of generator (or load).
$P^{max}_{kc}, P^{min}_{kc}$: maximum and minimum rating of transmission element; for lines we assume $P^{max}_{kc} = P^{min}_{kc}$. $P^{min}_{kc}$ has a positive value.
$R^{+}_{gct}, R^{-}_{gct}$: maximum ramp rate in the up and down direction for generator (or load) except in the startup period.
$R^{s}_{g}$: maximum ramp rate for the start up period for generator (or load).

$c_{gt}$: variable cost of production for generator (or value of load); generally $c_{gt} > 0$.
$c_{kt}$: variable cost of operating transmission element; generally $c_{kt} = 0$
$cr^+_{gt}, cr^-_{gt}$: cost of ramp rate in the up and down direction for generator (or load).
$\rho_c$: probability of the state $c$.
$SU_{gt}$: startup cost for generator (or load); generally for generators $SU_{gt} > 0$.
$NL_{gt}$: no-load cost for generator or load.
$S_{kt}$: cost of switching of transmission element (due to equipment stress).
$B_k$: electrical susceptance of transmission element.
$N1_{ec}$: binary parameter that is 0 when element $e$ ($k$ or $g$) is the contingency in state $c$, and is 1 otherwise.
$UT_g$: minimum up time for generator (or load).
$DT_g$: minimum down time for generator (or load).
$T$: number of periods.

### 4.3 Model formulation

To save space we combine the presentation of the MIP and its derivative linear program in one formulation. In the derived linear program, the constraints with named dual variables are retained from the MIP and the binary variables are set to their optimal values (see O'Neill et al. [32]). The objective is to maximize the expected market surplus (benefits to society).

*Maximize*

$$EMS = \sum_t \sum_g \left[ -I_{gt}x_{gt} - \sum_c \rho_c (SU_{gt}v_{gt} + c_{gt}P_{gct} + cr^+_{gt}r^+_{gct} + cr^-_{gt}r^-_{gct} + NL_{gt}u_{gt}) \right]$$
$$+ \sum_t \sum_k - \left\{ I_{kt}y_{kt} + \sum_c \rho_c [S_{kt}sw_{kct} + c_{kt}(P^+_{kct} + P^-_{kct})] \right\} \tag{1}$$

*Phase angle constraints*

$$\theta^+_{kct} \le \theta^+ \qquad \alpha^+_{kct} \qquad \forall k, c, t \tag{2}$$
$$\theta^-_{kct} \le \theta^+ \qquad \alpha^-_{kct} \qquad \forall k, c, t \tag{3}$$
$$\theta^+_{kct} - \theta^-_{kct} = \theta_{mct} - \theta_{nct} \qquad \overline{\alpha}_{kct} \qquad \forall k, c, t \tag{4}$$
$$\theta^+_{kct}, \theta^-_{kct} \ge 0 \tag{5}$$

In this formulation we constrain the angle difference. An alternative formulation is to constrain the angles. These bounds can act as a proxy for various nonlinear constraints that are not modeled, and are included to ensure a reasonable result from the linearized model.

*Power bus (node) balance equations (Kirchhoff's first law)*

$$\sum_{k(.,n)} (P^+_{kct} - P^-_{kct}) - \sum_{k(n,.)} (P^+_{kct} - P^-_{kct}) + \sum_{g(n)} P_{gct} = 0 \qquad \lambda_{nct} \qquad \forall n, t, c \quad (6)$$

*Transmission Asset Flow Limits*

$$P^+_{kct} - P^{max}_{kc} z_{kct} N1_{kc} \leq 0 \qquad \eta^+_{kct} \qquad \forall k, c, t \quad (7)$$

$$P^-_{kct} - P^{min}_{kc} z_{kct} N1_{kc} \leq 0 \qquad \eta^-_{kct} \qquad \forall k, c, t \quad (8)$$

$$P^+_{kct}, P^-_{kct} \geq 0 \qquad \forall k, c, t \quad (9)$$

$$z_{kct} \in \{0, 1\} \qquad \forall k, c, t \quad (10)$$

A transmission element $k$ is open when it fails in contingency, $N1_{kc} = 0$, or is opened for reliability/economic reasons $z_{kct} = 0$. As a result, $P^+_{kct} = P^-_{kct} = 0$. We assume transmission switching is immediate, therefore, it can be used in a contingency to optimize the system. This is a realistic assumption in cases where SPS are in place for specific scenarios. The switching satisfies all reliability constraints since feasibility satisfies reliability.

*Kirchhoff's Second Law for AC Transmission Elements*

The susceptance, $B_k$, is negative in value. When $\theta^+_{kct}$ is positive, the line flow will be negative, i.e, $P^+_{kct}$ needs to be zero and $P^-_{kct}$ needs to be positive. We have:

$$B_k \left(\theta^+_{kct} - \theta^-_{kct}\right) - \left(P^+_{kct} - P^-_{kct}\right) = 0 \quad (11)$$

We then replace (11) with (12) and (13):

$$B_k \theta^-_{kct} + P^+_{kct} = 0 \quad (12)$$

and

$$B_k \theta^+_{kct} + P^-_{kct} = 0 \quad (13)$$

To incorporate transmission switching and line outages, we change the constraints to:

$$-B_k \theta^-_{kct} - P^+_{kct} - M_k(2 - z_{kct} - N1_{kc}) \leq 0 \qquad \mu^{+M}_{kct} \qquad \forall k \in AC, c, t \quad (14)$$

$$-B_k \theta^+_{kct} - P^-_{kct} - M_k(2 - z_{kct} - N1_{kc}) \leq 0 \qquad \mu^{-M}_{kct} \qquad \forall k \in AC, c, t \quad (15)$$

$$B_k \theta^-_{kct} + P^+_{kct} \leq 0 \qquad \mu^+_{kct} \qquad \forall k \in AC, c, t \quad (16)$$

$$B_k \theta^+_{kct} + P^-_{kct} \leq 0 \qquad \mu^-_{kct} \qquad \forall k \in AC, c, t \quad (17)$$

In contingency (or state) $c$, $N1_{kc} = 0$. When $z_{kt} = 0$, the transmission element is not in the network for economic reasons. In both situations $P^+_{kct} = P^-_{kct} = 0$. If $N1_{kc} = 1$ and $z_{kt} = 1$, the transmission element is in the network and the second law is enforced.

*Switching indicator variables.* We capture the cost of switching a transmission element from open to closed or closed to open from one period to the next. We include the following constraints to enforce the relationship between $z_{kct}$ the period to period switching status variable $sw_{kct}$:

$$z_{kct} - z_{kct-1} - sw_{kct} \leq 0 \qquad \mu^{sw+}_{kct} \qquad \forall k \in AC, c, t \tag{18}$$

$$z_{kct-1} - z_{kct} - sw_{kct} \leq 0, \qquad \mu^{sw-}_{kct} \qquad \forall k \in AC, c, t \tag{19}$$

$$sw_{kct} \geq 0 \qquad \forall k \in AC, c, t \tag{20}$$

*Direct Current Lines.* DC lines are modeled as a paired generator and load at the terminal buses in either direction. For ease of presentation we do not include line losses or AC-DC and DC-AC conversion losses, but could include a linear approximation of losses.

A new transmission element $k$ cannot be in the network in period $t$ unless the investment has been made in a prior period $t' \leq t$; therefore,

$$z_{kct} - \sum_{t' \leq t} y_{kt'} \leq 0 \qquad \kappa_{kct} \qquad \forall k, c, t \tag{21}$$

$$y_{kt} \in \{0, 1\} \qquad \forall k, t \tag{22}$$

Note that, at optimality, $\sum_t y_{kt} \leq 1$. That is, at most one $y_{kt}$ variable will be equal to one at optimality; this could be a constraint that is imposed in the model as it may help computationally, but it is not needed to ensure optimality. For existing assets, $I_{kt} = 0$ and $y_{k1} = 1$ or Eqs. (21) and (22) are dropped. Going forward costs like fixed O&M are ignored, but variable O&M is included in $c_{kt}$.

*Generator or Load Upper and lower bounds.* In contingency $c$ (when $N1_{gc} = 0$) or when the unit is not started up ($u_{gt} = 0$), $P_{gct} = 0$.

$$P_{gct} - P^{max}_{gct} N1_{gc} u_{gt} \leq 0 \qquad \beta^+_{gct} \qquad \forall g, c, t \tag{23}$$

$$-P_{gct} + P^{min}_{gct} N1_{gc} u_{gt} \leq 0 \qquad \beta^-_{gct} \qquad \forall g, c, t \tag{24}$$

$$u_{gt} \in \{0, 1\} \qquad \forall g, t \tag{25}$$

*Ramp rate constraints.* Ramp rate constraints limit the change in output from $t-1$ to $t$ separately in both directions.

$$P_{gct} - P_{gct-1} - r^+_{gct} \leq 0 \qquad \omega^+_{gct} \qquad \forall g, c, t \tag{26}$$

$$r^+_{gct} - R^+_{gct} u_{gt-1} - R^s_g v_{gt} \leq 0 \qquad \chi^+_{gct} \qquad \forall g, c, t \tag{27}$$

$$P_{gct-1} - P_{gct} - r^-_{gct} \leq 0 \qquad \omega^-_{gct} \qquad \forall g, c, t \tag{28}$$

$$r^-_{gct} \leq R^-_{gct} \qquad \chi^-_{gct} \qquad \forall g, c, t \tag{29}$$

$$r^+_{gct}, r^-_{gct} \geq 0 \qquad \forall g, c, t \tag{30}$$

In Eq. (27), $R_g^s$ gives a separate ramp rate during startup.

*Startup, Minimum Uptime, Shutdown and Minimum Downtime Constraints.*

$$v_{gt} - w_{gt} - u_{gt} + u_{g,t-1} = 0 \qquad \tau_{gt} \qquad \forall gt \tag{31}$$

$$-u_{gt} + \sum_{q=t-UTg+1} v_{gq} \leq 0, \qquad \forall g, t \in \{UT_g, \ldots, T\} \tag{32}$$

$$u_{gt} + \sum_{q=t-DTg+1} w_{gq} \leq 1, \qquad \forall g, t \in \{DT_g, \ldots, T\} \tag{33}$$

$$v_{gt} \in \{0, 1\} \qquad \forall gt \tag{34}$$

$$w_{gt} \in \{0, 1\} \qquad \forall g, t \tag{35}$$

A new generator or load $g$ cannot be in the network unless the investment has been made to construct and interconnect. Because generators have minimum up time and down time constraints, the run status, $u_{gt}$, is not available in a contingency; therefore, $u_{gt}$ does not have a $c$ subscript.

$$v_{gt} - \sum_{t' \leq t} x_{gt'} \leq 0, \qquad \xi_{gt} \qquad \forall k, t \tag{36}$$

$$x_{gt} \in \{0, 1\} \qquad \forall g, t \tag{37}$$

For existing assets, $I_{gt} = 0$, $x_{g1} = 1$, and $x_{gt} = 0$ for $t > 1$ and Eqs. (36) and (37) are dropped. Fixed going forward costs are ignored.

*Renewable Portfolio Constraints.* To include renewable portfolio constraints, we add the following parameters and constraints. $RC_t$ is the renewable portfolio capacity requirement in period $t$. This requirement can easily be made region or renewable specific. $GR$ is the set of generators $g$ that meet a specific renewable portfolio constraint. For the specific renewable portfolio generation requirement in period $t$, we add the constraint:

$$-\sum_{g \in GR} \sum_{t' \leq t} x_{gt'} P_{g0t}^{max} \leq -RC_t \qquad \upsilon_t \qquad \forall t \tag{38}$$

where $\upsilon_t$ is the marginal cost of meeting the portfolio in period $t$. Here, with parameter $P_{g0t}^{max}$, we refer to the nameplate capacity of the renewable resource. The $c = 0$ index represents the scenario where the renewable resource's maximum capacity is the manufacturer nameplate capacity. In general, $P_{g0t}^{max} = P_{gct}^{max}$ as we do not change the nameplate capacity by scenario for renewable resources. $RP_t$ is a specific renewable portfolio production requirement in period $t$. This requirement can easily be made region or renewable specific. For the specific renewable portfolio production requirement in period $t$, we add the constraint:

$$-\sum_{g\in GR}\sum_{c}\rho_c P_{gct} \leq -RP_t \qquad \psi_t \qquad \forall t \tag{39}$$

where $\psi_t$ is the marginal cost of meeting the production requirement from the portfolio in period $t$. From here on we will use the renewable portfolio production requirement since the usual approach and policy goal is too stimulate renewable production not capacity. In addition, capacity requirements usually have weak incentives for production. Of course, both constraints could be employed if desired.

*Environmental Constraints.* To include environmental constraints we add the following parameters, variables and constraints. $EL_t$ is the amount of a pollutant allowed in period $t$. This requirement can be made region and/or pollutant specific. $a_{gt}$ is the amount of a pollutant per MWh for generator or load $g$ in period $t$. We add the constraint:

$$\sum_{g}\sum_{c}\rho_c a_{gt} P_{gct} \leq EL_t \qquad \iota_t \qquad \forall t \tag{40}$$

where $\iota_t$ is the marginal cost of reducing a unit of the pollutant in period t.

We note for the environmental and renewable production constraints that we have chosen a formulation which enforces the requirements based on expected values of renewable production and emissions across all contingencies and states $c$. While our assumption here is that the minimum requirements (right hand side parameters) for these constraints are developed based on ex ante statistical analysis and expected values, it may also be reasonable to assume that these requirements are developed without regard to expected values. In the latter case, the constraints would need to be reformulated with the probability dropped from the left hand side and the constraints enforced over all $c$. For example, Eq. (40) would be re-written as: $\sum_g a_{gt} P_{gct} \leq EL_t,\ \forall t, c$ with the dual variable $\iota_{tc}$. Of course, this would add even more dimensions to the problem. Pre-processing the inputs to reduce the number of enforced constraints would be beneficial and almost certainly necessary (this is true in general of the constraint set for this model).

At this point we have formulated a very large and complex problem that is difficult to solve using current computing technology. This poses a challenge to the mathematical programming community of how to solve such models on a timescale that matches the needs of analysts and planners, and how to adapt and re-solve such models to explore scenarios. To calibrate the scale of this challenge we note that the current application of mixed integer linear programming in pricing and dispatch of electricity corresponds to stage 2 in this model, but without the complication of transmission switching which adds enormous complexity. Production cost models currently used to analyze the economic effects of transmission expansion simulate generator commitment, dispatch and pricing but do not attempt to optimize transmission expansion (likely due to computational complexity). Nevertheless, we next proceed to form a dual problem and present an economic analysis of the theoretical solution. This shows the relative tractability of sensitivity analysis for this problem.

### 4.4 Economic analysis using the dual problem

Sensitivity analysis can address many issues including sensitivity to data inputs, assumptions, approximations, and market power. The list is large and the analysis can be computationally intense. Duality analysis can make some analysis computationally simpler, practical and focused. For the purpose of economic analysis, stochastic MIP economic duality can be considered in two parts. First, a linear program can be formed with the binary variables fixed at their optimal values. For linear programs, marginal economic information is almost computationally free from the dual program. Second, a set of binary variables in the MIP can be changed, the MIP is then resolved, and the change in the objective function then gives the expected incremental value for that set of binary variables. For complete analysis, all combinations of binary variables need to be considered, but this is practically impossible for all but small programs. For practical reasons we need to find analysis that is more cost beneficial. Consequently, duality analysis of the linear program can be helpful in marginal analysis and in choosing the binary variables for the incremental analysis. Hence, we form the 'economic' dual for sensitivity analysis.

The set of feasible solutions to a MIP may be nonconvex, but for a fixed set of binary variables, the resulting feasible solution set is either empty or a convex polytope. By setting the integer variables to their values in the optimal or best solution found, the resulting problem is a linear program and the resulting dual is well defined and yields economic information about the solution. The linear program is optimal with respect to the fixed integer variables and the LP, its dual and the MIP objective function have the same optimal value [32].

Once the integer values are fixed, some constraints and variables become redundant creating many choices for formulating the linear program and its dual. We will choose one that yields an intuitive economic interpretation. The following analysis holds even if the feasible MIP solution is not a global optimum and the solution is only a local optimal solution for a fixed set of binary variables.

We form a linear program by replacing the binary constraints with constraints setting the variables equal to their optimal (or best) values and assigning each a dual variable:

$$z_{kct} = z^*_{kct} \qquad \delta_{kct} \qquad \forall k, c, t \tag{41}$$

$$y_{kt} = y^*_{kt} \qquad \pi_{kt} \qquad \forall k, t \tag{42}$$

$$v_{gt} = v^*_{gt} \qquad \sigma_{gt} \qquad \forall g, t \tag{43}$$

$$w_{gt} = w^*_{gt} \qquad \gamma_{gt} \qquad \forall g, t \tag{44}$$

$$x_{gt} = x^*_{gt} \qquad \varphi_{gt} \qquad \forall g, t \tag{45}$$

If $z_{kt} = 0$ or if $N1_{kc} = 0$, Eqs. (14) and (15) are not binding constraints, and we drop them from the formulation. If $N1_{kc} = 1$ and $z_{kt} = 1, 2 - z_{kt} - N1_{kc} = 0$ and constraints (14) and (15) along with (16) and (17) form equality constraints. Constraints (14)-(17) can then be replaced by (46), or (47) and (48).

$$B_k(\theta^+_{kct} - \theta^-_{kct}) - P^+_{kct} + P^-_{kct} = 0 \qquad \mu_{kct} \qquad \forall k \in AC, c, t, z^*_{kt} = 1, N1_{kc} = 1 \tag{46}$$

$$B_k\theta^-_{kct} + P^+_{kct} = 0 \qquad \mu^+_{kct} \qquad \forall k \in AC, c, t, z^*_{kt} = 1, N1_{kc} = 1 \tag{47}$$
$$B_k\theta^+_{kct} + P^-_{kct} = 0, \qquad \mu^-_{kct} \qquad \forall k \in AC, c, t, z^*_{kt} = 1, N1_{kc} = 1 \tag{48}$$

We choose (47) and (48). Since Eqs. (21), (31)–(33) and (36) are redundant in the linear program, we also drop these equations. We now write the dual of the above linear program. The objective of the dual program is to minimize the opportunity costs of meeting the optimal plan, which is equal to maximizing the benefits to society.
*Dual: Minimize*

$$MSD = \theta^+ \sum_t \sum_k \sum_c (\alpha^+_{kct} + \alpha^-_{kct}) + \sum_t \sum_g \left[ \sum_c (R^-_{gct}\chi^-_{gct}) + x^*_{gt}\varphi_{gt} + v^*_{gt}\sigma_{gt} + w^*_{gt}\gamma_{gt} \right]$$
$$+ \sum_t \sum_k (y^*_{kt}\pi_{kt} + z^*_{kt}\delta_{kt}) - \sum_t (RP_t\psi_t + RC_t\upsilon_t - EL_t\iota_t) \tag{49}$$

*Angle value constraints.*

$$\alpha^+_{kct} + \overline{\alpha}_{kct} + B_k\mu^-_{kct} \geq 0 \qquad \theta^+_{kct} \qquad \forall k \in AC, c, t, z^*_{kt} = 1, N1_{kc} = 1 \tag{50}$$
$$\alpha^-_{kct} - \overline{\alpha}_{kct} + B_k\mu^+_{kct} \geq 0 \qquad \theta^-_{kct} \qquad \forall k \in AC, c, t, z^*_{kt} = 1, N1_{kc} = 1 \tag{51}$$
$$\sum_{k(.,n)} \overline{\alpha}_{kct} - \sum_{k(n,.)} \overline{\alpha}_{kct} = 0 \qquad \theta_{nct} \qquad \forall k \in AC, c, t, z^*_{kt} = 1, N1_{kc} = 1 \tag{52}$$

If $\theta^+_{kct} > 0, \alpha^+_{kct} + \overline{\alpha}_{kct} = -B_k\mu^-_{kct}$. If $\theta^-_{kct} > 0, \alpha^-_{kct} - \overline{\alpha}_{kct} = -B_k\mu^+_{kct}$.

*Flowgate value constraints for AC transmission.*

$$\lambda_{mct} - \lambda_{nct} + \eta^+_{kct} + \mu^+_{kct} \geq -\rho_c c_{kt} \quad P^+_{kct} \quad \forall k \in AC, c, t, z^*_{kct} = 1, N1_{kc} = 1 \tag{53}$$
$$\lambda_{nct} - \lambda_{mct} + \eta^-_{kct} + \mu^-_{kct} \geq -\rho_c c_{kt} \quad P^-_{kct} \quad \forall k \in AC, c, t, z^*_{kct} = 1, N1_{kc} = 1 \tag{54}$$

*Flowgate value constraints for DC transmission.*

$$\lambda_{mct} - \lambda_{nct} + \eta^+_{kct} \geq -\rho_c c_{kt} \qquad P^+_{kct} \qquad \forall k \in DC, c, t, z^*_{kct} = 1, N1_{kc} = 1 \tag{55}$$
$$\lambda_{nct} - \lambda_{mct} + \eta^-_{kct} \geq -\rho_c c_{kt} \qquad P^-_{kct} \qquad \forall k \in DC, c, t, z^*_{kct} = 1, N1_{kc} = 1 \tag{56}$$

The susceptance values, $\mu^+_{kct}$ and $\mu^-_{kct}$ do not appear in the equations for DC lines.

*Flowgate investment constraints.*

$$\begin{aligned} &-P^{max}_{kc}N1_{kc}\eta^+_{kct} - P^{min}_{kc}N1_{kc}\eta^-_{kct} + \delta_{kct} + \kappa_{kct} + \mu^{sw+}_{kct} - \mu^{sw+}_{k,t+1,c} \\ &\quad + \mu^{sw-}_{k,t+1,c} - \mu^{sw-}_{kct} = 0 \\ &\quad z_{kct} \qquad \forall k \in AC, c, t \end{aligned} \tag{57}$$

$$-P^{max}_{kc}N1_{kc}\eta^+_{kct} - P^{min}_{kc}N1_{kc}\eta^-_{kct} + \delta_{kct} + \kappa_{kct} = 0 \qquad z_{kct} \qquad \forall k \in DC, c, t \tag{58}$$

$$-\mu_{kct}^{sw+} - \mu_{kct}^{sw-} \geq -\rho_c S_{kt} \qquad sw_{kct} \qquad \forall k \in AC, c, t \tag{59}$$

$$\pi_{kt} - \sum_{c} \sum_{t' \geq t} \kappa_{kct'} = -I_{kt} \qquad y_{kt} \qquad \forall k, t \tag{60}$$

*Operating constraints for generation and load elements.*

$$\lambda_{nct} - \beta_{gct}^{-} + \beta_{gct}^{+} + \omega_{gct}^{+} - \omega_{gct}^{-} - \omega_{gc,t+1}^{+} + \omega_{gc,t+1}^{-} - \rho_c(a_{gt}\iota_t + \psi_t) = -\rho_c c_g$$
$$P_{gct} \qquad \forall g, c, t \tag{61}$$

*Startup constraints.*

$$\xi_{gt} + \tau_{gt} + \sigma_{gt} - \sum_{c} R_g^s \chi_{gct}^{+} = -SU_{gt} \qquad v_{gt} \qquad \forall g, t \tag{62}$$

$$-\tau_{gt} + \gamma_{gt} = 0 \qquad w_{gt} \qquad \forall g, t \tag{63}$$

$$\sum_{c}(-P_{gct}^{max} N1_{gc}\beta_{gct}^{+} + P_{gct}^{min} N1_{gc}\beta_{gct}^{-}) - \tau_{gt} + \tau_{g,t+1} - R_{g,c,t+1}^{+}\chi_{g,c,t+1}^{+} = -NL_{gt}$$
$$u_{gt} \qquad \forall g, t \tag{64}$$

*Ramp constraints*

$$-\omega_{gct}^{+} + \chi_{gct}^{+} \geq -\rho_c cr_g^{+} \qquad r_{gct}^{+} \qquad \forall g, c, t \tag{65}$$

$$-\omega_{gct}^{-} + \chi_{gct}^{-} \geq -\rho_c cr_g^{-} \qquad r_{gct}^{-} \qquad \forall g, c, t \tag{66}$$

*Investment constraints*

$$\varphi_{gt} - \sum_{t' \geq t} \xi_{gt'} = -I_{gt} \qquad x_{gt} \qquad \forall g, t \tag{67}$$

$$\alpha_{nct}^{+}, \alpha_{nct}^{-}, \eta_{kct}^{+}, \eta_{kct}^{-}, \beta_{gct}^{+}, \beta_{gct}^{-}, \psi_t, \omega_{gct}^{+}, \omega_{gct}^{-}, \chi_{gct}^{+}, \chi_{gct}^{-}\mu_{kct}^{sw+}, \mu_{kct}^{sw-}, \kappa_{kct},$$
$$\xi_{gt}, \upsilon_t, \iota_t \geq 0 \tag{68}$$

If $z_{kct}^{*} = 1$, $N1_{kc} = 1$ and $P_{kct}^{+} \neq P_{kct}^{-}$, then $\eta_{kct}^{+}\eta_{kct}^{-} = 0$.
If $u_{gt}^{*} = 1$, $N1_{gc} = 1$ and $P_g^{-} \neq P_g^{+}$ : $\beta_{gct}^{+}\beta_{gct}^{-} = 0$. $r_{gct}^{+}, r_{gct}^{-} = 0$. $\omega_{gct}^{+}\omega_{gct}^{-} = 0$. $\chi_{gct}^{+}\chi_{gct}^{-} = 0$.

## 4.5 Economic analysis of the stochastic investment market

An analysis of 'post investment' economics in the dual problem for a unit commitment and transmission switching problem is presented in O'Neill et al. [6]. Here we examine the investment economics for the above auction formulation.

For transmission elements in the optimal plan, $y_{kt}^{*} = 1$ and for some t and state c, if the transmission asset is in the optimal topology, $z_{kct}^{*} = 1$ for period t and state c, and

| $\kappa_{kct} =$ | $-\delta_{kct}$ | $+(P_{kc}^{max} N1_{kc}\eta_{kct}^{+} + P_{kc}^{min} N1_{kc}\eta_{kct}^{-})$ | $-\mu_{kct}^{sw+} + \mu_{kct}^{sw-}$ | $-\mu_{k,c,t+1}^{sw-} + \mu_{k,c,t+1}^{sw+}$ |
|---|---|---|---|---|
| Profit in *t* and *c* | System value of being in the system in *t* and *c* | linear incremental value (*LIV*) of another unit of capacity | Incremental value of switching from open to closed or closed to open in *t* | Incremental value of switching from open to closed or closed to open in *t+1* |

(69)

Let $LIV_{kct} = P_{kc}^{max} N1_{kc}\eta_{kct}^{+} + P_{kc}^{min} N1_{kc}\eta_{kct}^{-}$. Since $P_{kc}^{max}, N1_{kc}, \eta_{kct}^{+}, P_{kc}^{min}$, $\eta_{kct}^{-} \geq 0$, then $LIV_{kct} \geq 0$. Expected incremental switching value in period *t*, $\delta_{kct}$, may be either positive or negative. If $\kappa_{kct} < 0$, it appears to be uneconomic. However, since the system is optimal, due to the nonconvexities of the market, and as a result of switching and the interaction of AC lines via susceptances, the value of the flowgate is in combination with other elements positive and adds to the market surplus. In a market where *LIV* is the settlement value as it is in FTR markets or some flowgate markets, without a call option for commitment, the transmission owner would remove it from the network. The minimum payment to the owner to keep the asset in the network is $\kappa_{kct} > 0$. Today the RTO or ISO system operator usually has an option call on the asset in return for a cost-of-service payment.

Let $\underline{\kappa}_{kt} = \sum_c z_{kct}^{*}\kappa_{kct}$. Multiplying by $z_{kct}^{*}$ and summing over the contingencies *c*, expected profit in *t* can be stated as

$$\underline{\kappa}_{kt} = -\sum_c z_{kct}^{*}\delta_{kct} - \sum_c z_{kct}^{*}\left(P_{kc}^{max} N1_{kc}\eta_{kct}^{+} + P_{kc}^{min} N1_{kc}\eta_{kct}^{-}\right) + \sum_c z_{kct}^{*}\left(\mu_{kct}^{sw+} - \mu_{k,c,t+1}^{sw+} + \mu_{k,c,t+1}^{sw-} - \mu_{kct}^{sw-}\right) \quad (70)$$

If the transmission investment occurs in period *t*, $y_{kt}^{*} = 1$, total expected profitability from the investment $\underline{\pi}_{kt}$ can be defined as

$$\underline{\pi}_{kt} = \underline{\kappa}_{kt} + \underline{\kappa}_{k,t+1} + \cdots + \underline{\kappa}_{kT} - I_{kt} \quad (71)$$

If $\pi_{kt} > 0$ and $y_{kt}^{*} = 1$, for some t, a risk-neutral investor would undertake the investment *k* stimulated by its positive expected profitability. If $\pi_{kt} < 0$ and $y_{kt}^{*} = 1$, for some *t*, the linear value is negative, but since the investment is optimal, there is a coalition of market participants who would be willing to make up the shortfall. The reasoning is simple. If investment *k* is removed, absent non-uniqueness, the market surplus falls; therefore, there is a coalition of market participants who receive less benefits, i.e., they would be better off with the investment. Consider all paths in the cut set from *m* to *n*. In what follows we will assume a unique solution. Assume the flow is from *n* to *m* and $\lambda_{nct} < \lambda_{mct}$. Let $path(n, m)$ be a path from *n* to *m*; this can be a direct path, i.e. one transmission line, or a path involving multiple transmission assets.

From adding over $k \in path(n, m)$, the intermediate $\lambda$'s cancel and $\sum_{k \in path(n,m)} \lambda_{mct} - \lambda_{nct} = \lambda_{mct} - \lambda_{nct} = \sum_{k \in path(n,m)} \eta^{+}_{kct} + \mu^{+}_{kct} + \rho_c c_k t$.

If for all $k \in path(n, m)$, $0 < P^{+}_{kct} < P^{max}_{kc}$, and $c_{kt} = 0$, then $\eta^{+}_{kct} = 0$, and $\lambda_{mct} - \lambda_{nct} = \sum_{k \in path(n,m)} \mu^{+}_{kct}$. This result is extended by superpositioning to any cut set, multiple paths and any number of elements in the series.

Let $\eta_{kct} = \eta^{+}_{kct} + \eta^{-}_{kct}$ and $P_{kct} = P^{+}_{kct} + P^{-}_{kct}$. If the transmission investment in DC cable k occurs in period $t$, $y^{*}_{kt} = 1$, total expected profitability of the investment, $\pi_{kt}$ can be defined as

$$\pi_{kt} = \eta_{kct} P_{kct} + (\eta_{k,c,t+1})(P_{k,c,t+1}) + \cdots + \eta_{kcT} P_{kcT} - I_{kt} \tag{72}$$

If $\pi_{kt} > 0$ and $y^{*}_{kt} = 1$, for some $t$, the investment would be stimulated by its expected profitability. If $\pi_{kt} < 0$ and $y^{*}_{kt} = 1$, for some $t$, the investment is optimal even though the linear value is negative. The reasoning is the same as previously. If investment $k$ is removed the market surplus falls, therefore, there is a coalition of market participants who receive less benefits who would be better off subsidizing the investment.

If the generation and/or load investment occurs in period $t$, $x^{*}_{gt} = 1$, Eq. (67) can be restated and interpreted as the sum of the discounted expected profits in each period over the post investment horizon.

$$\varphi_{gt} = \xi_{gt} + \xi_{g,t+1} + \cdots + \xi_{gT} - I_{gt} \tag{73}$$

If $\varphi_{gt} > 0$ and $x^{*}_{gt} = 1$, for some $t$, the investment would be stimulated by its expected profitability. If $\varphi_{gt} < 0$ and $x^{*}_{gt} = 1$, for some $t$, the linear value is negative, but since the investment is optimal, there is a coalition of market participants who would be willing to make up the shortfall. Again, if investment $g$ is removed from the market surplus falls, then, there is a coalition of market participants who receive less benefits who would be better off subsidizing the investment.

If the renewable portfolio constraint is only a production constraint, the dual constraint for production (61) is modified as follows. Since most renewables have near zero production costs and low emissions, we can set $c_g = 0$ and $a_{gt} = 0$. Since variable energy resources are the lowest costs units on the system and ramping is driven by the state of nature, we can drop the ramp rate constraints and, therefore, $\omega^{+}_{gct} = \omega^{-}_{gct} = 0$, $\forall t$. For variable or renewable energy resource $g$, (61) becomes $\lambda_{nct} - \beta^{-}_{gct} + \beta^{+}_{gct} - \rho_c \psi_t = 0$, $\forall c, t$. Summing over $c$ and $t$, we obtain $\psi_g = \sum_c \sum_t \rho_c \psi_t = \sum_c \sum_t (\lambda_{nct} - \beta^{-}_{gct} + \beta^{+}_{gct})$. $\psi_g$ can be interpreted as the expected unit production subsidy necessary to induce renewable or other generation into the market. A similar analysis can be undertaken for capacity.

*Environmental Constraints.* The environmental constraint is a production constraint. The dual constraint for production is modified as follows (61). For non-renewable resources, with $c_g > 0$ and $a_{gt} > 0$ (assuming a resource that is not ramp constrained), (61) becomes $\lambda_{nct} - \beta^{-}_{gct} + \beta^{+}_{gct} - \rho_c a_{ct} \iota_t = -\rho_c c_g$, $\forall c, t$. Summing over $c$ and $t$, we obtain $\iota_g = \sum_c \sum_t \rho_c a_{ct} \iota_t = \sum_c \sum_t (\rho_c c_g + \lambda_{nct} - \beta^{-}_{gct} + \beta^{+}_{gct})$. $\iota_g$ can be interpreted as the expected unit additional costs of generation. Summing over $g$, we obtain $\sum_g \iota_g = \sum_g \sum_c \sum_t \rho_c a_{ct} \iota_t = \sum_g \sum_c \sum_t (\rho_c c_g + \lambda_{nct} - \beta^-_{gct} + \beta^+_{gct})$.

## 5 Summary, conclusions and additional work

In this paper we presented a process for planning, formulated a transmission planning model, introduced sensitivity analysis techniques, and presented several approaches to allocation of transmission costs and rights. The planning model is a multi-period N-1-reliable unit commitment, transmission switching and investment model. This model poses a challenge to the optimization community of building computational tools to facilitate decision making by providing analysts and planners better information in reasonable time. Beyond optimization algorithms, modelling tools for exploring of the tradeoff between modelling accuracy and computational complexity are also called for. Sensitivity analysis can help examine model assumptions and can help find relaxations that could reduce the computational burden. Future work would involve identifying the necessary approximations and demonstrating the feasibility of the modeling approach computationally. Several different levels of equivalenced models defined by the ability to solve them on the available hardware and software could be developed, ranging from rough approximations that solve in hours to more granular approahces that are given up to several days to solve. Evaluating additional details, such as the incorporation of different loss approximation techniques, could be included as part of identifying necessary approximations. The results and computational performance from incorporating varying levels of detail could be compared and evaluated to determine both the acceptable and necessary levels of approximation.

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