# Stochastic Unit Commitment with Topology Control Recourse for Power Systems with Large-Scale Renewable Integration

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Abstract-- In this paper, we model topology control through transmission switching as a recourse action in the day-ahead operation of power systems with large-scale renewable generation resources. We prove that transmission switching could only reduce the linear objective value of direct current optimal power flow (DCOPF) when congestion exists. However, we also show, through a simple example, that unit commitment cost could be reduced by transmission switching even in the absence of congestion. To solve the stochastic unit commitment with topology control within reasonable computational time, we proposed a heuristic that first decomposes a practical system into zones and then solves the problem for each zone in parallel. The benefit of topology control recourse is demonstrated on a network representing the Central European System. We compare the costs of the network with different loading and renewable generation conditions. The cost reduction of the test system can reach 3.34% with heavy load and large-scale renewable generation while in a single zone the cost reduction can be above 7%.

Index Terms—Renewable generation integration, stochastic unit commitment, topology control, transmission switching.

#### I. NOMENCLATURE

Indices and Sets:

- *i*, *j* Buses (nodes).
- Time period. t
- Scenario. S
- Conventional generator. g
- Wind generator. w
- Ζ. Zone.
- G Set of generators.
- GS Set of slow generators.
- GFSet of fast generators.
- Ν Set of buses.
- М Set of lines.
- Ζ Set of zones.
- N(i)Set of buses connected to bus i.
- $N_z$ Set of buses in zone z
- S Set of scenarios.
- Т Set of time periods,  $T = \{1, 2, ..., 24\}$ .
- GWSet of wind generators.

#### Parameters:

- $B_{ii}$ The susceptance of line *ij*.
- $D_{i,t}$ The demand on bus i at time t.

- $D_{i,s,t}^{net}$ The net load on bus i at time t in scenario s.
- Cost(revenue) of power import(export) on line ij.  $\tau_{ii}$
- The probability of scenario s.  $\pi_{s}$
- The start-up cost of generator g.  $h_{g}$
- $k_{g}$ The no-load cost of generator g.
- $C_g$ The fuel cost of generator g.
- The penalty cost of load shedding at bus *i*.  $\rho_i$

Variables:

- The commitment of generator g at time t in scenario  $\mathcal{U}_{g,t,s}$
- The start-up indicator of generator g at time t in  $\sigma_{_{g,t,s}}$ scenario s.
- $P_{g,t,s}$ The production level of generator g at time t in scenario s.
- Reserve of generator g at time t.  $\gamma_{g,t}$
- Active power flow on line *ij* at time *t* in scenario *s*.  $F_{ii,t,s}$
- $\theta_{i,t,s}$ Voltage angle of bus *i* at time *t* in scenario *s*.
- On-off status of line *ij* at time *t* in scenario *s*.  $r_{ij,t,s}$
- Load shedding on bus *i* at time *t* in scenario *s*.  $L_{i,t,s}$

#### **II. INTRODUCTION**

ENEWABLE generation is the fastest growing K contributor to the electricity portfolio across the world. In the United States, different states adopt different Renewable Portfolio Standards (RPS). Taking California as an example, the RPS program requires investor-owned utilities, electric service providers, and community choice aggregators to increase procurement from eligible renewable energy resources to 33% of total procurement by 2020. However, the uncertain and variable nature of renewable generation brings new challenges to power system operations. In contrast with conventional generations, the renewable generation output is constrained by uncertain natural resources such as wind and solar. Hence, renewable generation cannot be controlled so as to generate the desired amount of electricity. Moreover, the forecast accuracy of such generation resources is relatively low up to few hours ahead of the delivery time. Due to the limitation of large-scale storage in current power systems, demand and supply must be matched instantaneously. To serve the demand reliably with the presence of variability and uncertainty of renewable generations, more flexibility, which can be obtained from generation, transmission, and demand, is required.

Power systems are designed to have some inherent level of flexibility in operations to balance supply and demand at all times, such that unpredictable load changes and unexpected conventional generation failures would not affect the reliability of the system. Conventionally, the operational flexibility requirements are met by the deploying of a set of generators with enough headroom and ramping capability. For systems with large-scale renewable generation, resource uncertainty must be explicitly accounted for in the day ahead unit commitment. This can be done by optimizing the unit commitment as a two-stage stochastic programming problem. Such optimization typically differentiates between slow ramping resources that must commit before the uncertainty in renewable resources is realized, and flexible resources that provide recourse capability in response to diverse realizations of renewable generation. Recourse actions serve as hedging mechanisms that reduce the need for reserves provided by the early commitment of slow ramping generators. With largescale renewables integration, increased amounts of reserves through the commitment of more flexible conventional generation resources are required. Such a solution is expensive and could undermine the economic and environmental objectives of deploying renewable resources. In this paper, we focus on the use of topology control through switching on/off transmission lines, which has received little attention as a means for recourse actions in response to uncertain renewable resources. Practical issues such as postswitching stability must be validated offline and are out of the scope of the paper. If the switching of lines undermines the stability of the system, we could constrain the line not to be switched and resolve the problem. As for N-1 reliability criteria, it should be noted that in the context of stochastic optimization such criteria is replaced by incorporating generators and line contingencies into the probabilistic scenarios such a formulation allows us to consider multiple contingencies simultaneously. Such contingencies can be mitigated by transmission switching recourse actions along with the response to the renewables generation uncertainty. We may also introduce post optimization processing to evaluate the robustness of the solution with respect to specific contingencies and eliminate switching actions that do meet the criterion. In this paper, we only focus on optimizing the recourse switching decisions with respect to renewable generation uncertainties and defer the consideration of other contingencies to future work.

Both deterministic models and stochastic models have been studied for unit commitment with large-scale renewable integration. Sioshansi et al. [1] presented a deterministic unit commitment model in which explicit constraints on reserves and import limit are imposed. The excess generation capacity required by those constraints can be utilized to balance the supply and demand in the system when contingencies occur, or renewable supply has fluctuations. Another way to formulate this problem is to adopt stochastic programming models where the uncertainty of renewable generation is modeled as weighted scenarios, and reserve requirements are modeled endogenously [2–4]. In stochastic models, the uncertainty is usually represented as a set of scenarios. To reduce the computation complexity, various scenario reduction techniques have been proposed in [5–7]. To solve stochastic unit commitment problems, researchers have adopted and developed decomposition algorithms include Lagrangian relaxation [4], augmented Lagrangian methods [3] and progressive hedging [2]. In general, Lagrangian methods are sensitive to the selection of the parameters. Progressive hedging has been proven to be a rigorous algorithm for convex problems and convergence is guaranteed [8]. However, for the unit commitment problem that is discrete, cycling phenomenon has been observed in some test instances as presented in [9]. When we include switching decisions, the symmetry caused by the presence of identical parallel lines might lead to more redundant computation [10]. These solution techniques are developed for solving stochastic unit commitment without transmission switching. If switching decisions are included, then scalable techniques or heuristics are required to solve the problem efficiently. There is also extensive literature on chanceconstrained formulations [11] and robust optimization models [12] of the day-ahead scheduling problems for power systems with variable renewable resources. Here, we focus only on the two-stage stochastic programming formulation.

Transmission lines are traditionally considered as uncontrollable static assets in the operations of power systems. However, in practice, system operators can and do change the topology of the transmission network as postcontingency control actions. The switching of transmission lines incurs no additional cost other than possible wear of the breakers, which is typically small comparing to the potential benefits. In this paper, we do not consider the cost of switching. The idea of transmission switching has been studied for decades. Transmission lines are switched on/off as preventive or corrective actions to enhance system security by relieving overload conditions [13]. In [14], the authors utilized transmission switching as corrective actions to deal with both load violation and voltage violation. In the most recent paper [15], the authors compared the results of contingency in day-ahead unit commitment, with and without transmission switching. They showed that corrective transmission switching helped not only in post-contingency situations but also in achieving N-1-1 reliability. Transmission switching has also been studied as a method to harness flexibility from existing transmission systems to reduce the system operating cost in the context of direct current optimal power flow (DCOPF) [16-18]. In DCOPF, transmission switching makes it possible to choose the best system topology so that the power generation is optimized on that topology. In Section III we will show that transmission switching reduces the system cost by relieving congestion. In [19], the authors extended the optimal switching in real-time operations into the context of alternative current optimal power flow (ACOPF). They proposed a two-level iterative framework, in which a second-order cone programming is solved to provide candidate optimal switching solution in the upper level and then the solution is screened to achieve AC feasibility in the lower level. In [20], the authors proposed a model that considers both short-circuit current limit constraints and N-1 security constraints in optimal transmission switching problem. In the proposed algorithm, N-1 criteria is checked after the optimization in each iteration. In their model they include the cost of switching lines. In the numerical test, results show that switching cost might

influence the switching decision but not the performance of the proposed model. To solve the problem efficiently, researchers have developed heuristics to obtain near-optimal switching decisions. Ruiz et al. proposed fast heuristics for OPF with topology control instead of solving MIP directly [21, 22]. Transmission switching has also been shown to reduce the investment cost in power system expansion. In [23], the authors formulate the line capacity expansion problem as a two-stage stochastic programming problem, and switching decisions are made in the second stage with other operational decisions, while the investment decisions are made in the first stage. Their results showed that with transmission switching, the network could be augmented cheaper with respect to total cost including both investment cost and operational cost. In day-ahead operations of power systems without renewable generation [24], the authors studied how transmission switching will change the optimal deterministic unit commitment as well as the optimal cost. The switching decisions and deterministic security-constraint unit commitment decisions are optimized iteratively. In this deterministic problem, the cost reduction is not as substantial as in optimal transmission switching on the IEEE 118 system. For day-ahead scheduling problems with large-scale renewable integration, more cost reduction is expected since the flexibility provided by transmission switching can mitigate the uncertainty of renewable generation which may lead to more efficient commitment decisions. We have shown in our previous paper [25] that transmission switching as a recourse action in response to realized uncertainty of intermittent renewable resources could mitigate such adverse variability and improve unit commitment efficiency in IEEE 118 system.

In this paper, we focus on topology control recourse through the switching of transmission lines in two-stage stochastic unit commitment. The objective of this paper is to demonstrate that topology control as a recourse action may provide benefits by mitigating the variability of wind generation and reduce the system operating cost in commercial-scale power systems. The remainder of the paper is organized as follows. In section III, we compare the role of transmission switching in optimal power flow and unit commitment. In section IV, we propose a scheme that decomposes an interconnected commercial system into zones. In section V, we provide a demonstration study based on a network representing the Central European system. And section VI concludes the paper.

## III. TRANSMISSION SWITCHING, OPTIMAL POWER FLOW AND UNIT COMMITMENT

Transmission switching in a deterministic optimal power flow and unit commitment settings for IEEE test cases has been proved very effective in reducing the operating cost [16, 26]. Switching on /off transmission lines can divert flows in the system and relieve congestion or potential congestions in the system. Hence, generation decisions are able to be adjusted optimally reduce operating to cost. In this section, we will prove that in a single period DCOPF, transmission switching can only reduce the objective function value when congestion is present. However, we will also show, through an example that in a multi-period unit commitment, this is no longer the case due

to the discrete nature of the optimization problem. In fact, transmission switching can enlarge the feasible region of commitment decisions and hence reduce total cost by reducing "potential" congestions.

#### A. Transmission Switching and Optimal Power Flow

In a DCOPF setting, we formally prove that if there are no line congestions, changing the topology brings no benefits to the objective.

The output of generators is denoted as a vector  $P_G \in \mathbb{R}^N$ . When there is no generation connected to a particular bus, the corresponding component is zero. The voltage angles of buses are represented using vector  $\mathbf{\theta}_N$ . The active power flow of transmission lines is represented as  $\mathbf{F}_M$ .

The optimal power flow (OPF) problem is defined as:  $(OPF) \min \mathbf{c}_G^T \mathbf{P}_G$ 

$$s.t.\begin{bmatrix} I_{|N| \times |N|} & 0_{|N| \times |N|} & A_{|N| \times |M|} \\ 0_{|M| \times |N|} & B_{|M| \times |N|} & -I_{|M| \times |M|} \end{bmatrix} \begin{bmatrix} \mathbf{P}_{G} \\ \mathbf{\theta}_{N} \\ \mathbf{F}_{M} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_{N} \\ 0 \end{bmatrix}$$
$$\mathbf{P}_{G}^{\min} \leq \mathbf{P}_{G} \leq \mathbf{P}_{G}^{\max}$$
$$-\mathbf{F}_{M}^{\max} \leq \mathbf{F}_{M} \leq \mathbf{F}_{M}^{\max}$$

where  $|\cdot|$  represents the cardinality of a set,  $A_{|N| \times |M|}$  is the incidence matrix. Matrix *B* is a sparse matrix with non-zero components:

$$B_{l,i} = \frac{1}{x_l}, B_{l,j} = -\frac{1}{x_l}$$

where  $x_l$  is the impedance of line l starting from bus iending at bus j. The optimal solution of *OPF* is  $\overline{\mathbf{x}}^T = \left[\overline{\mathbf{P}}_G^T \quad \overline{\mathbf{\theta}}_N^T \quad \overline{\mathbf{F}}_M^T\right]$ . Due to the optimality of  $\overline{x}$ , there is no feasible direction  $\mathbf{d} = \left[\mathbf{d}_{P_G}^T \quad \mathbf{d}_{\theta_N}^T \quad \mathbf{d}_{F_M}^T\right]^T$  such that  $\mathbf{c}_G^T \mathbf{d}_G < 0$ . When there is transmission switching, the operating cost will be less than or equal to the operating cost of *OPF*.

Let us consider an *OPF* with optimal production  $\overline{\mathbf{P}}_{G}$ . There is no congestion, i.e. the line flow capacity constraints are not binding. The optimal production level  $\mathbf{P}_{G}^{*}$  of transmission switching problem *OTS* can be obtained by solving the following problem:

 $(OTS) \min \mathbf{c}_G^T \mathbf{P}_G$ 

$$\begin{split} s.t. \begin{bmatrix} I_{|N| \times |N|} & 0_{|N| \times |N|} & A_{|N| \times |\tilde{M}|} \\ 0_{|\tilde{M}| \times |N|} & B_{|\tilde{M}| \times |N|} & -I_{|\tilde{M}| \times |\tilde{M}|} \end{bmatrix} \begin{bmatrix} \mathbf{P}_{G} \\ \mathbf{\theta}_{N} \\ \mathbf{F}_{\tilde{M}} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_{N} \\ 0 \end{bmatrix} \\ \mathbf{P}_{G}^{\min} &\leq \mathbf{P}_{G} \leq \mathbf{P}_{G}^{\max} \\ -\mathbf{F}_{\tilde{M}}^{\max} \leq \mathbf{F}_{\tilde{M}} \leq \mathbf{F}_{\tilde{M}}^{\max} \end{split}$$

where  $\tilde{M}$  is the set of transmission lines after switching on/off transmission lines. The *OTS* is essentially a *OPF* solved on a network with a different set of transmission lines.

Assume  $\mathbf{c}_G^T \mathbf{P}_G^* < \mathbf{c}_G^T \overline{\mathbf{P}}_G$ . Let  $\mathbf{d}_{P_G} = \mathbf{P}_G^* - \overline{\mathbf{P}}_G$ . The feasible direction of *OPF* satisfies:

$$B_{|M| \times |N|} \mathbf{d}_{\theta_N} = \mathbf{d}_{F_M}$$
$$\mathbf{d}_{P_G} + A_{|N| \times |M|} \mathbf{d}_{F_M} = 0$$

We will get

$$\mathbf{d}_{P_{G}} = B' \mathbf{d}_{\theta_{N}}$$

where *B*' is the coefficient matrix of DC power flow equation and it is of rank |N|-1. We can delete the last row and the last column. We can first take the inverse of the reduced matrix to get  $\mathbf{d}_{q_{|N|-1}}$  using  $\mathbf{d}_{P_G}$  and then find the last component of  $\mathbf{d}_{\theta_N}$ . Since the line flow capacity constraints in *OPF* is not binding, there exists a feasible direction for *OPF* satisfying  $\mathbf{c}_G^T \mathbf{d}_{P_G} < 0$ , which contradicts the optimality of  $\mathbf{\bar{P}}_G$ when switching is not allowed. Thus, if there is no congestion in optimal power flow, transmission switching will not benefit the system. Based on this result, heuristics could be developed by monitoring only a subset of transmission lines and target the relieving line flow congestions in the OPF solution.

#### B. Transmission Switching and Unit Commitment

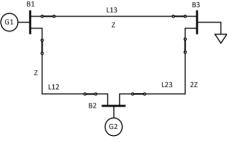


Fig. 1 3-bus system

While the above result may seem intuitive, this is presented here in order to highlight the contrast with the multi-period unit commitment optimization where transmission switching could be beneficial even in the absence of congestion, as illustrated through the following simple example. It is shown that "potential" line flow congestions may lead to an optimal commitment decision, such that if we fix the binary commitment decisions, no congestions can be observed in the optimal solution. Switching on/off some of the lines can still reduce the cost.

Consider a 2-period deterministic unit commitment problem for a 3-bus system with 2 units as shown in Fig. 1. The impedance of lines is also depicted on the figure.

Generator	G1	G2
Start-up Cost	100	100
No-load Cost	70	150
Production Cost (per Unit of Demand)	10	5
Capacity	30	30
Ramping Rate (up and down)	5	10

TABLE I Generator parameters of the 3-bus system

TABLE II Line flow capacities of the 3-bus system

Line	L12	L23	L13
Flow Capacity	8	30	30

We consider only two time periods. The demand in the two periods is 18 and 24. Other related parameters of generators and transmission lines are listed in Table I and Table II, for brevity, the parameters are all measured as the ratio with respect to some base value.

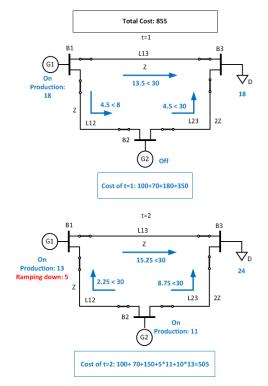


Fig. 2. UC solution without transmission switching

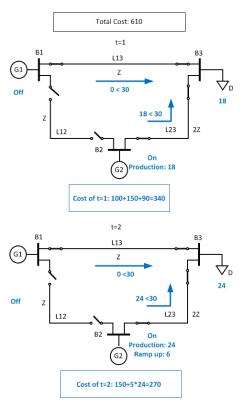


Fig. 3. UC solution with transmission switching

The results are depicted in Fig. 2 and Fig. 3. Though there is no line congestion observed in the optimal solution of UC without transmission switching, the cost can be reduced in both periods by switching off L12. From the optimal solution of UC with transmission switching, we can see that for both time periods it is optimal to commit only G2. Without switching, L12 would be congested if we only had G2. Hence, only G1 is committed in t1. Since the binary decision is changed, the potential congestion could not be observed. An important implication highlighted by this example, which is significant for the main problem explored in this paper, is that heuristics based on relieving congestions on specific lines cannot be directly applied in this case. For stochastic unit commitment with transmission switching recourse, the problem is even more complicated.

#### IV. FRAMEWORK FOR SOLVING STOCHASTIC UNIT COMMITMENT WITH TRANSMISSION SWITCHING RECOURSE

Regional transmission organizations, like ENTSO-E in Europe and PJM in the US, coordinate the transaction of electric power through different regions or zones subject to different regulatory frameworks. Taking ENTSO-E as an example, each zone in the interconnected system has its own transmission system operator (TSO). Cross-border electricity exchange is cleared in the day-ahead market without considering renewable generation. Inspired by the current operation of this large-scale power system, we propose a framework to solve stochastic unit commitment with transmission switching recourse (TCSUC) for commercialscale interconnected power systems, in which TSOs of each zone co-optimize generator commitment and dispatch decisions as well as transmission line switching decisions.

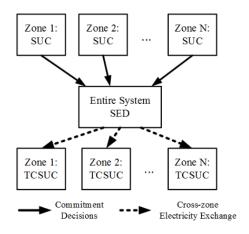


Fig. 4. Framework for solving TCSUC for commercial-scale interconnected system

In the proposed framework shown in Fig. 4, the problem we solve in each step is a two-stage stochastic programming problem. Each zone first solves a stochastic unit commitment (SUC) problem without transmission switching. In SUC, the inter-zone flows are modeled as electricity imports and exports. The generators are divided into two sets: fast generators and slow generators. The commitment of slow generators is the first-stage decision. The commitment of fast generators is determined after the realization of renewable generation. The SUC can be formulated as:

$$(SUC) \min \sum_{i \in T} \left( \sum_{s \in S} \pi_s \left( \sum_{g \in G_s} \left( h_g \sigma_{g,s,s} + k_g u_{g,s,s} + c_g P_{g,s,s} \right) + \sum_{g \in GS} c_g P_{g,s,s} \right) + \sum_{i \in N} \rho_i L_{i,s,s} \right) + \sum_{g \in GS} \left( h_g \sigma_{g,s} + k_g u_{g,s} \right) \right) + \sum_{i \in T} \sum_{s \in S} \pi_s \left( \sum_{i \in N_s} F_{ij,s,s} \tau_{ij} + \sum_{i \in N_s} F_{ij,s,s} \tau_{ij} \right)$$
  
$$s.t. \quad \left( \mathbf{u}_{GS}^z, \mathbf{\sigma}_{GS}^z \right) \in \Delta_{GS}$$
$$\left( \mathbf{u}_{GF,s}^z, \mathbf{P}_{G,s}^z, \mathbf{F}_s^z, \mathbf{F}_s^z, \mathbf{L}_s^z \right) \in \Delta_{GF} \left( \mathbf{W}_s^z, \mathbf{u}_{GS}^z \right), \forall s \in S$$

In SUC, the last term of the objective represents the cost/revenue for exchanging electricity with other zones. The vector  $\mathbf{F}_{s}^{z}$  represents line flow within the zone z while  $\mathbf{F}_{s}^{c}$ 

represents line flows between zones. For brevity, we do not list all constraints for SUC. The first constraint represents the on/off transition, minimum up time, and minimum down time constraints of slow units.  $\Delta_{GF}(\mathbf{W}_s^z, \mathbf{u}_{GS}^z)$  represents the feasible set of the second stage decisions that depend on the renewable generation scenarios and the first-stage decisions.

All constraints are linear. The detailed formulation can be found in our previous paper [25].

The binary commitment decisions are submitted to the interconnected system coordinator. The coordinator has access to the model and the data of each zone. It then solves a stochastic economic dispatch (SED) problem in the dayahead market for the entire system. The SED is a two-stage stochastic programming problem. The cross-border flows are first stage decisions forced by non-anticipativity constraint since this value is settled in day-ahead before the realization of uncertainty; generation dispatches are second stage decisions:

(SED) min 
$$\sum_{t \in T} \sum_{s \in S} \pi_s \left( \sum_{g \in GF_z} c_g P_{g,t,s} + \sum_{g \in GS} c_g P_{g,t,s} + \sum_{i \in N} \rho_i L_{i,t,s} \right)$$
  
s.t.  $\left( \mathbf{P}_{G,s}^Z, \mathbf{F}_s^Z, \mathbf{L}_s^Z \right) \in \Delta \left( \mathbf{W}_s^Z, \mathbf{F}^C \right), \forall s \in S, z \in Z$ 

Noted that here we have  $\Delta(\mathbf{W}_s^z, \mathbf{F}^c)$  defining the feasible region. In  $\Delta(\mathbf{W}_s^z, \mathbf{F}^c)$ , decision variable  $\mathbf{F}^c$  is the first-stage decision and it does not depend on scenarios. It is equivalent to using  $(\mathbf{P}_{g,s}^z, \mathbf{F}_s^z, \mathbf{L}_s^z) \in \Delta(\mathbf{W}_s^z, \mathbf{F}_s^c)$  for all scenarios and zones and then forcing  $\mathbf{F}_s^c = \mathbf{F}^c$  for any scenario *s*. In SED, we have market clearing constraints, line flow constraints, line flow capacity constraints, generation capacity constraints, and ramping constraints. The SED is a stochastic linear programming problem. Different time periods are coupled by the ramping constraints. Different scenarios are coupled by the cross-border line flows.

Finally, the cross-border flows are broadcast to zones, and each TSO solves TCSUC with the cross-border flows fixed. For each renewable generation scenario, the cross-border flows are same as the solution in SED. The formulation of TCSUC is similar as that of SUC other than line flow constraints and flow capacity constraints. The two constraints are modeled as:

$$-\mathbf{M}_{ij}\left(1-r_{ijJ,s}\right) \leq F_{ijJ,s} - B_{ij}\left(\theta_{iJ,s} - \theta_{jJ,s}\right) \leq \mathbf{M}_{ij}\left(1-r_{ijJ,s}\right),$$
  
$$\forall i, j \in N_{z}, t \in T, s \in S$$
(1)

$$-r_{i_{j,s,s}}F_{i_j}^{\max} \le F_{i_{j,s,s}} \le r_{i_{j,s,s}}F_{i_j}^{\max}, \forall i, j \in N_z, t \in T, s \in S$$
(2)

We allow switching for all lines within each zone. Crossboarder lines are always on. Constraint (1) represents the modified linear approximation of Kirchhoff's current law. The parameter  $M_{ij}$  in this constraint has to be greater or equal to  $|B_{ij}| \max(|\theta_i - \theta_j|)$ , so that if a line is off, the voltage angles of the two buses are no longer related. We want  $M_{ij}$  to be as small as possible to generate efficient cuts. In normal operating states of power systems, the voltage angle difference between connected buses is below 5°. Using the topology information of the system, we can estimate  $\max(|\theta_i - \theta_j|)$  and a sufficiently small value of  $M_{ij}$ . Constraint (2) states that if a line is off, the line flow is zero. If a line is not switched off, the flow on it should be within the flow limit.

The proposed model decomposes the large-scale mixed integer programming problem into sub-problems with fewer decision variables and significantly reduce the complexity of solving the problem. The sub-problems can be solved in parallel, and the decomposition can be fit into the coordination framework of interconnected power systems. Moreover, solutions to SUC serve as warm-starts of TCSUC, which helps reduce the solution time.

Since we fix the cross-border flows between zones and only allow the switching of lines within a zone, the solution to the decomposed model is sub-optimal. For the central European system that we used in the numerical test with 10 scenarios, there are over 400,000 binary variables without decomposition. Even if we use scenario based decomposition method such as Progressive Hedging, there are around 80,000 binary variables in each sub-problem. For an interconnected practical power system with thousands of buses, such heuristics that reduce the complexity of solving the problem is essential. Given that realtime coordination between zones is still limited in the current European market, we think it is reasonable to model power exchanges between zones as dayahead pre-fixed values instead of decisions, and it is appropriate to leave the switching decisions in the zonal level detailed transmission configuration where system information is accessible.

### V. NUMERICAL TEST

In this section, we conduct numerical tests on a network representing the Central European system [26]. We compare the costs for each zone with and without topology control recourse in two settings of renewable generation and loading conditions. In the first setting (Case 1), we use base values for both loads and renewable generations. In the second setting (Case 2), we increase the load by 10% and increase the renewable generation by 5% to create more congestions in the system.

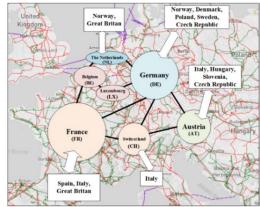


Fig. 5. Central European System

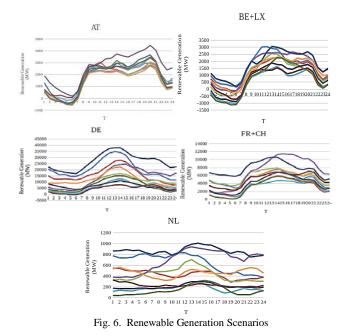
#### A. Test System and Data Description

There are 679 buses, 667 conventional units, 1036 transmission lines and 1437 renewable units in our test system. The interconnections between different countries within and outside the Central European system are shown in Fig. 5.

There are 7 countries in the network. Detailed information on the grid of those countries is listed in Table III. Among all 7 countries, Germany has the largest number of buses, lines, and units. The peak load of Switzerland and Luxembourg is much larger than the maximum generation capacity (Max. Gen. Cap.). We need to combine the two countries with other interconnected countries as zones to balance the load and generation. There are 9 lines connecting Switzerland and France and 5 lines connecting Switzerland and Germany. Switzerland is connected more tightly with France than Germany. When we solve the problem, we take France and Switzerland as a single zone. Similarly, we also consider Belgium and Luxembourg as a zone. In our numerical tests, we decompose the system into 5 zones.

TABLE III System Zonal Information

	AT	BE	СН	DE	FR	LX	NL
Buses	36	24	47	228	317	3	24
Lines	42	23	76	312	518	2	26
Fast Units	11	25	4	94	22	0	19
Slow Units	25	45	5	254	108	1	46
Peak Load (MW)	8044.9	1.3e4	7328	65018	69043	839	13959
Max. Gen. Cap. (MW)	7656.8	1.7e4	4335.1	1.1e5	9.0e4	375	24690



We select 10 renewable generation scenarios from 120 simulated scenarios using historical data. Since our goal is to justify the benefit of topology control recourse in stochastic unit commitment, we want scenarios with high variance, so that they cover extreme realizations of renewable generation. The total renewable generation in each scenario of a day is plotted in Fig. 6. From the figure, we can see that renewable generation scenario can be as much as roughly 10 times the lowest renewable generation represents pumped hydroelectric storage. Taking the two largest zones FR+CH and DE as examples, the maximum penetration of renewable generation in the second setting among all scenarios and all time periods

is 17.1% in FR+CH and 60.5% in DE.

#### B. Test Results and Analysis

The test results of the two cases are listed in Table IV and Table V. The solver we used is CPLEX. The optimality gap of SUC is set to be 0.5% for all zones. The optimality gap of TCSUC is set to be 4% for DE and FR+CH, and 2% for the other three zones to ensure all sub-problems can be solved within the similar amount of time. The lower bound of TCSUC is from solving a relaxation of the problem. Solutions with high optimality gap are feasible solution whose objective value is greater than or equal to the optimal one. We did not choose a uniform gap for different zones in TCSUC but the result is still comparable since the higher optimality gap induce a lower bound for cost reduction. The larger the optimality gap is in TCSUC, the more conservative the cost saving is. From the results, we can see that with topology control recourse, the total cost of the system will be reduced by 0.2944 million euros in Case 1 and 1.8794 million euros in Case 2.

TABLE IV Case 1 Test Results

	SUC (MEUR)	TCSUC (MEUR)	Cost Saving (MEUR)
AT	3.2257	3.2123	0.0134
BE+LX	3.2130	3.2119	0.0011
DE	15.1121	15.0078	0.1043
FR+CH	13.5106	13.3395	0.1711
NL	4.1957	4.1912	0.0045
Total	39.2571	38.9627	0.2944

From the results, we can see that the total cost of Case 2 is higher than that of Case 1, but the percentage cost saving is 3.34% with topology control recourse which is much higher than 0.75% in the first case. The zone FR+CH has the largest cost saving with transmission switching recourse in both cases. In contrast, no cost saving is observed in the zone BE+LX in Case 2 and the cost saving in the zone representing NL is close to zero in Case 1.

	SUC (MEUR)	TCSUC (MEUR)	Cost Saving (MEUR)
AT	7.0057	6.8244	0.1813
BE+LX	6.2083	6.2083	0.00
DE	14.2089	14.0540	0.1549
FR+CH	17.3961	16.0753	1.3478
NL	10.5475	10.3793	0.1682
Total	55.3665	53.5141	1.8521

TABLE V Case 2 Test Results

In Case 2, the load on each bus is increased by 10%. Not only more generation is required to balance the demand, but also more congestion is created. Due to the congestion, more expensive units have to be scheduled to meet the demand. Hence, the total cost increase is high. Moreover, with 5% more renewable generation, more flexible units with higher costs need to be deployed to mitigate the variability without topology control recourse. Thus, topology control plays a more important role in Case 2. In the following analysis when we compare the cost savings in different zones, we will focus on Case 2.

To understand why the percentage cost saving in FR+CH is above 7% while that of BE+LX is zero, we will examine the loading conditions, ramping capabilities, congestions of the two zones. We also define other metrics to analyze the

results. The net load ramping (NLR) requirement is defined as:

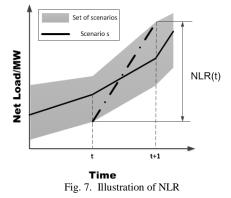
$$NLR^{Up}(t) = \max_{s \in S} \left( D_{s,t+1}^{net} \right) - \min_{s \in S} \left( D_{s,t}^{net} \right)$$

$$NLR^{Down}(t) = \max_{s \in S} \left( D_{s,t}^{net} \right) - \min_{s \in S} \left( D_{s,t+1}^{net} \right)$$

$$NLR(t) = \max \left\{ NLR^{Down}(t), NLR^{up}(t) \right\}, \forall t \in T$$

$$NLR = \max_{t \in T} NLR(t)$$
(3)

Fig. 7 provides a graphical illustration on how NLR is defined. The NLR measures the variability of renewable generations. Moreover, it is designed to capture the extremes among different scenarios, which reflects the extreme ramping requirements in the second stage that the operators need to consider when the first stage decisions are made. In the two-stage stochastic unit commitment, the first stage commitment decisions need to accommodate such variability in the second stage among different scenarios. For a system with high NLR value, if the ramping capacity of fast units is small, more first-stage commitment decisions will be cut-off, and the number of feasible solutions will be smaller. Thus, the higher the value of NLR is, the fewer the feasible slow generator commitments there are.



We also calculated the congestion rate for zone z defined as:

$$CR_{z} = \frac{1}{\left(\#T\right)\left(\#M_{z}\right)} \sum_{s \in S} \pi_{s} \sum_{t \in T} \sum_{i, j \in N_{z}} \mathbf{1}\left(\left|F_{ij,t,s}\right| = F_{ij}^{\max}\right)$$
(4)

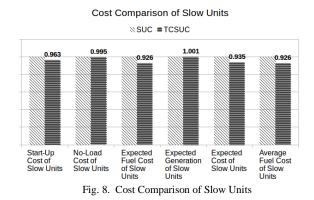
where # represents the cardinality of a set,  $\mathbf{1}(\cdot)$  is the indicator function,  $|\cdot|$  is the absolute value of a variable, and  $F_{i_{j,t,s}}$  is the line flow of the stochastic unit commitment without topology control recourse. This quantity represents the average percentage of lines congested per time period. It attempts to quantify how congested the network is.

Statistics of the two zones are listed in Table 6. We can see that the NLR of BE+LX is much higher than the ramping capacity of slow units while the NLR of FR+CH is closed to the ramping capacity of slow generators. Thus, in BE+LX, the variability of the renewable generations is mitigated by fast units. Moreover, the CR of FR+CH is much lower than that of BE+LX. The zone of BE+LX is more congested than FR+CH. Topology control recourse can change the commitment schedule through reducing potential congestions. However, if there are too many lines congested in a network, by switching on/off lines might not enlarge the feasible set of the first stage commitment decisions to provide a better solution.

TABLE VI Loading, generation and congestion statistics of BE+LX and FR+CH

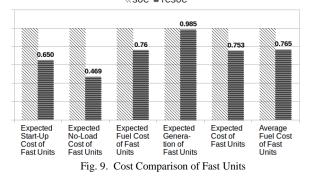
	BE+LX	FR+CH
Total Max Cap. of Slow Generators	13899.6	84952
Total Max Cap. of Fast Generators	3086.5	9730
Total Slow Ramping Cap.	1424.4	10034.7
NLR	2381.4	12809.4
Max. Net Load	12451.3	77447.2
Min. Net Load	9682.8	50682.3
CR	0.1916	0.0254

The comparison of detailed cost information in zone FR+CH is shown in Fig. 8 and Fig. 9. To compare different cost components of slow units and fast units, we scale each component through dividing it by the corresponding value of SUC. The values in the figures represent the cost component in TCSUC corresponding to that of SUC. From Fig. 8, we can see that by including topology control as a recourse action, the first stage commitment decisions have been altered so that both start-up cost and no-load cost have been reduced. Moreover, almost the same amount of generation from slow units is dispatched in the second stage. But the average fuel cost of TCSUC is lower than that of SUC. The expected fuel cost of slow units is decreased with topology control recourse. Similarly, the expected start-up cost expected no-load cost, and expected fuel cost are all reduced. Around 1.5% less fast generation is dispatched in TCSUC. That reduction in fast generation is covered by slow units with cheaper fuel costs. From the results, we can see that with topology control recourse in stochastic unit commitment, we can utilize the flexibility provided by switching on/off transmission lines to mitigate the variability introduced by renewable generation.



Cost Comparison of Fast Units

⊗SUC ≡TCSUC



VI. CONCLUSION AND FUTURE WORKS

We have studied modeling topology control through transmission switching as a recourse in a two-stage stochastic unit commitment model for power systems with large-scale renewable generation. We analyzed how the switching decisions could affect the commitment decisions and the dispatching decisions in OPF and unit commitment. To solve TCSUC for practical system efficiently, we also proposed a decomposition heuristic. Numerical tests conducted on a network representing the Central European System demonstrate that with topology control recourse, the expected operating cost will be reduced. The flexibility provided by topology control allows the commitment of cheaper units in both stages in the stochastic unit commitment problem. But such flexibility is limited by other conditions of the system. We observed that, for heavily congested systems, the operating cost could not be reduced significantly by purely introducing topology control recourse.

For future research, system contingencies traditionally monitored trough N-1 security criteria should be included in the probabilistic scenarios so that they are accounted for in the first-stage decision of stochastic unit commitment. Further work is also needed for developing efficient scenario based decomposition algorithms, accounting for switching cost and preventing cycling.

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